

Annual Talks 2025

February 03 – 04, 2025

Schedule and Abstracts of Talks

**School of Mathematics
Tata Institute of Fundamental Research**

Title of Talks

Mahan Mj	<i>Commensurations</i>
Balarka Sen	<i>Positive scalar curvature metrics on circle bundles over 4-manifolds</i>
Anantadulal Paul	<i>Counting plane curves with tangency</i>
Rashmita Hore	<i>Connectivity bounds of quadrature domains</i>
Eknath Ghate	<i>Reductions of Galois representations using the Iwahori mod p LLC</i>
Soumyadip Sahu	<i>Derived Hecke action on the trivial cohomology of division algebra</i>
Tanmay Deshpande	<i>The geometric Whittaker model, central sheaves and vanishing sheaves</i>
Ritankar Nath	<i>On the K-Theory and Homology Theories of Quotient Deligne-Mumford stacks</i>
Amartya Muthal	<i>Forward and backward dynamics of UPG outer automorphisms.</i>
Viswanathan S	<i>On Certain Combination Theorems</i>
Yaan Bugeaud	<i>On the decimal expansion of e</i>

Abstracts

Monday, 3 February 2025 (10:30-11:30)

Speaker : Mahan Mj
Title : Commensurations

A group G is said to commensurate a subgroup H if, for all $g \in G$, the intersection $H^g \cap H$ has finite index in both H and H^g , where H^g denotes the conjugate of H by g . The commensuration action of G on H can be analyzed dynamically. In this talk, we will explore a range of theorems and conjectures in this area, starting with the foundational work of Margulis and progressing to contemporary developments.

Monday, 3 February 2025 (11:45-12:15)

Speaker : Balarka Sen
Title : Positive scalar curvature metrics on circle bundles over 4-manifolds

In this talk, I will report on some findings pertaining to positive scalar curvature (psc) metrics on (trivial and non-trivial) circle bundles over 4-manifolds. The highlight of our work are the following results: (a) We give examples of enlargeable manifolds in all dimensions greater than 3, which admit (non-trivial) circle bundles with total space admitting psc metrics. This answers a question of M. Gromov. (b) For a simply connected closed 4-manifold M , we show $M \times S^1$ admits a psc metric if and only if M possesses a smooth structure admitting a psc metric. This addresses the “ S^1 -stability” conjecture of J. Rosenberg, in dimension 4. (c) Let M be a simply connected closed 4-manifold such that no smooth structure on M admits a psc metric. Then, if a Riemannian band $M \times [-1, 1]$ is uniformly psc, we show that the width of the band is bounded above by a dimensional constant. This addresses the “band-width inequality” conjecture of M. Gromov, in dimension 4.

The main techniques used are: symplectic geometry for (a), minimal surface methods and four dimensional surgery theory for (b), (c). While (b),(c) were known in dimensions 2,3,5,6 without altering the underlying smooth structure, that is false in dimension 4 due to obstructions from Seiberg-Witten theory. Thus, our results in this sense are optimal. We also discuss versions of (b) and (c) for non-simply connected 4-manifolds, which are part of on-going work. These results are joint works with Aditya Kumar (JHU).

Monday, 3 February 2025 (02:00-02:30)

Speaker : Anantadulal Paul
Title : Counting plane curves with tangency

In this talk, we will focus on counting curves of degree d in \mathbb{CP}^2 with tangencies. We will discuss the enumeration of plane degree d curves (possibly singular) tangent to a given line at multiple points with arbitrary order of tangency. Our interpretation of tangency as the limit of two transverse intersections was the main concept we used. Next, we will discuss the enumeration of plane curves with two nodes and plane tacnodal curves using tangency.

Monday, 3 February 2025 (02:45-03:15)

Speaker : Rashmita Hore
Title : Connectivity bounds of quadrature domains

Quadrature domains are domains in the Riemann sphere satisfying a finite node quadrature identity. The problem of topology of quadrature domains is a crucial part of the inverse problem in potential theory and the study of Hele-Shaw flows with applications to mathematical physics and fluid dynamics. The first results in the direction of connectivity of quadrature domains were obtained by B. Gustafsson in the paper Quadrature identities and the Schottky double. In the paper Topology of quadrature domains by S. Y. Lee and N. G. Makarov, sharp upper bounds for the connectivity of quadrature domains are established, in terms of the number of nodes and their multiplicities in the quadrature identity. We employ the Aharonov-Shapiro characterization of quadrature domains in terms of the Schwarz function and dynamics of Schwarz reflection maps to provide an alternative proof of the connectivity bounds of quadrature domains and strengthen some of the bounds in the paper by Lee and Makarov.

Monday, 3 February 2025 (03:30-04:30)

Speaker : Eknath Ghate

Title : Reductions of Galois representations using the Iwahori mod p LLC

We first recall the mod p Local Langlands Correspondence using Iwahori induction. We then use the Iwahori mod p LLC to compute the reductions of all 2-dimensional semi-stable representations of the Galois group of \mathbb{Q}_p of weights up to $p + 1$.

We show that the reduction varies through an alternating sequence of irreducible and reducible representations. In the reducible cases, we also determine the tricky constants appearing in the unramified characters completely. This result allow us to complete our proof of our zig-zag conjecture in the crystalline world.

In principle, our method works for all weights. In particular, it lets us go beyond the earlier glass ceiling of weight $p - 1$ which occurs in the deep work of Breuil-Mézard and Guerberoff-Park using methods from integral p -adic Hodge theory.

This is joint work with Anand Chitrao.

Monday, 3 February 2025 (05:00-05:30)

Speaker : Soumyadip Sahu

Title : Derived Hecke action on the trivial cohomology of division algebra

This talk aims to describe a generalization of A. Venkatesh's structure theorem for the action of derived Hecke algebra on the Hecke trivial cohomology of division algebras over number fields. In particular, we show that the Hecke trivial cohomology of a division algebra is a free module for the action of the strict derived Hecke algebra. Moreover, the strict derived Hecke algebra possesses a rational form that preserves the canonical rational structure on the cohomology during the derived Hecke action. The main ingredients in our improvement are a careful study of the congruence classes in the torsion cohomology of the arithmetic manifold and my new result on the reduction map in the K-theory of the ring of integers in number fields.

Monday, 4 February 2025 (09:30-10:30)

Speaker : Tanmay Deshpande

Title : The geometric Whittaker model, central sheaves and vanishing sheaves

Let G be a reductive algebraic group over an algebraically closed field. In this talk I will introduce the bi-Whittaker equivariant Hecke category of G . This monoidal category is a geometric analogue of the endomorphism algebra of an induced representation from a non-degenerate multiplicative character of a maximal unipotent subgroup. A classical result of Gelfand-Graev states that such induced representations are multiplicity free and contain “most” of the irreducible representations of G .

I will discuss the equivalence of the above bi-Whittaker Hecke category to a certain category of central sheaves on the maximal torus as well as the relationship of these categories with a certain category of vanishing conjugation equivariant sheaves on G . I will relate these results to the above result of Gelfand-Graev and the theory of character sheaves on reductive groups. This is based on joint work with R. Bezrukavnikov.

Monday, 4 February 2025 (11:00-11:30)

Speaker : Ritankar Nath

Title : On the K -Theory and Homology Theories of Quotient Deligne-Mumford stacks

A fundamental problem in the algebraic K -theory of stacks (or of equivariant K theory) is to determine how far it is from the ordinary K theory of schemes or algebraic spaces, especially that of a smooth (or étale) cover of the underlying stack. The much celebrated Atiyah Segal Completion theorem deals with this problem and we have various versions of the theorem for different comparisons between the K theories. We will discuss one such version which compares the algebraic K -theory of a quotient Deligne-Mumford stack with the Čech hypercohomology of the K -theory spectra of its inertia stack. This version of the completion problem has occurred previously in the works of Edidin-Graham(2005) and Krishna-Sreedhar(2017). We will then extend this to the Hochschild homology and its cyclic counterparts, under suitable circumstances. This is based on a joint work with Amalendu Krishna.

Monday, 4 February 2025 (11:45-12:15)

Speaker : Amartya Muthal
Title : Forward and backward dynamics of UPG outer automorphisms.

Bestvina, Feigh and Handel introduced the notion of improved relative train tracks to show that under forward iteration of Unipotent Polynomially Growing (UPG) outer automorphisms on the outer space, thought of as the space of free simplicial trees, points converge to a very small simplicial tree. In this talk I will show how the methods introduced by them can be applied to obtain the same result for backwards iteration and that the limit in these two cases is the same.

Monday, 4 February 2025(02:00-02:30)

Speaker : Viswanathan S
Title : On Certain Combination Theorems

Efforts in combining two different dynamical systems has shed light on deformation spaces of Kleinian groups. In this talk we report on a work in progress, wherein we present a single framework to combine several Kleinian groups (uniformizing genus 0 orbifolds) with hyperbolic rational maps. This naturally gave us examples of algebraic correspondences residing on higher genus Riemann surfaces.

Monday, 4 February 2025(02:45-03:45)

Speaker : Yaan Bugeaud
Title : On the decimal expansion of e

It is commonly expected that e , $\log 2$, $\sqrt{2}$, π , among other classical numbers, behave, in many respects, like almost all real numbers. For instance, one believes that their decimal expansion contains every finite block of digits from $\{0, \dots, 9\}$. We are very far away from establishing such a strong assertion. However, there has been some recent progress in that direction and we are now able to show that the decimal expansions of irrational algebraic numbers, of non-zero rational powers of e , of $\log(1 + \frac{1}{a})$ (provided that the integer a is sufficiently large), among other examples, are not ‘too simple’, in a suitable sense. We survey these results and give some ideas of their proofs, for which we take a point of view from combinatorics on words.