An introduction to the p-adic Langlands correspondence for $\mathrm{GL}_2(\mathbb{Q}_\mathrm{p})$

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1 Abstract

The goal of this series of lectures is to present Colmez's construction and study of the p-adic Langlands correspondence for $\mathrm{GL}_2(\mathbb{Q}_p)$, as well as more recent developments of the subject. The theory has been simplified and refined since Colmez's work, and we will try to give an overview of these advances. The main goal of the course will be the classification of absolutely irreducible unitary admissible Banach space representations of $\mathrm{GL}_2(\mathbb{Q}_p)$, for all primes p (this is work in progress by Colmez, Paskunas and the lecturer). We will also try to explain some of the applications of the theory, to the Fontaine-Mazur and Breuil-Mézard conjectures, as well as to Iwasawa theory of p-adic Galois representations and p-adic L-functions (depending on time).

2 Lectures

- Lecture 1: phi-gamma modules and $\mathrm{GL}_2(\mathbb{Q}_p)$-equivariant sheaves on the projective line, consequences of Schur's lemma and infinitesimal actions.
- Lecture 2: Locally analytic and locally algebraic vectors in p-adic Banach representations of $\mathrm{GL}_2(\mathbb{Q}_p)$, applications to the Fontaine-Mazur and Kisin conjecture.
- Lecture 3: Paskunas' finiteness theorem.
- Lecture 4: The classification theorem for absolutely irreducible unitary admissible Banach representations of $\mathrm{GL}_2(\mathbb{Q}_p)$.
- Lecture 5: Further applications: Iwasawa theory and p-adic L-functions.