

ON THE MOD p -LANGLANDS PROGRAM FOR $\mathbf{GL}_2$

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This course is aimed at giving an overview on the p -modular Langlands program for $\mathbf{GL}_2$ over a p -adic field, with a particular emphasis to the construction of local automorphic objects and their finer properties.

After recalling the classification results for $\mathbf{GL}_2$, we focus our attention to the structure of the automorphic representations for $\mathbf{GL}_2(\mathbf{Q}_p)$, especially in restriction to compact subgroups, with application to the local/global compatibility of the p -modular correspondence. We subsequently analyze Colmez's constructions through (φ, Γ) -modules, giving explicit examples on irreducible objects and insisting on the compatibility with the p -adic correspondence (following Berger).

We end the course by further investigating the phenomenology on local representations for $\mathbf{GL}_2$, giving a detailed account on Schraen's results on their presentation and Paskunas's results on their restriction to a Borel subgroup. Time permitting, we will study some of the parameters appearing in Breuil-Paskunas representations, linking them to p -adic Hodge theoretical data in the context of the local/global compatibility conjectures.

The program of the lectures will be approximately:

- Lecture 1 Classification results on p -modular representations of $\mathbf{GL}_2$.
- Lecture 2 Structure results for p -modular representations of $\mathbf{GL}_2(\mathbf{Q}_p)$.
- Lecture 3 Study of invariant spaces for mod p -representations appearing in the mod p -Langlands correspondence and global applications.
- Lecture 4 The Colmez's functor mod p , following Berger's approach, and some explicit examples.
- Lecture 5 On the presentation of supersingular representations, following Schraen.

The main references for the course are the following:

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