Global methods in the p-adic Langlands program

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This course is about global methods in the p-adic Langlands program, with a particular focus on the cohomology of curves.

In the first part, we will describe the situation concerning the group $\mathrm{GL}_2(\mathbb{Q}_p)$ where the situation is (almost) completely understood. We will define the completed p-adic topology of a tower of modular curves and explain how it can be described using the p-adic Langlands correspondence for $\mathrm{GL}_2(\mathbb{Q}_p)$ (Emerton).

In the second part we will be interested in very partial extensions of these results to other fields. In particular, if a p-adic correspondence exists for the group $GL_2(F)$ we can expect that it gives informations on completed cohomology of tower of geometric objects such as Shimura curves. We will focus here on a result which is the consequence of works of Breuil and Emerton-Gee-Savitt. These results imply that some supersingular mod p representations of $\mathrm{GL}_2(F)$ with F an unramified extension of $\mathbb{Q}_p$, as constructed by Breuil and Paskunas, appear in the mod p cohomology of towers of Shimura curves.

The program of the lectures will be approximatively the following.
- Lecture 1 Description of the p-adic Langland correspondence.
- Lecture 2 Completed cohomology, deformation rings.
- Lecture 3 The local-global compatibility theorem.
- Lecture 4 Cohomology of Shimura curves and link with supersingular representations of $\mathrm{GL}_2(F)$.
- Lecture 5 Multiplicity one theorem and proof of Breuil conjecture on lattices in the cohomology of Shimura varieties.

Références

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