

Annual Talks 2024
School of Mathematics, TIFR

21–23 February, 2024

Colloquium

Dipendra Prasad

TITLE: Character theory at special elements

ABSTRACT: There seems to be a relationship between irreducible characters of finite groups or of compact Lie group G at special elements in G and the dimension of irreducible representations of the centraliser of that element in G . For example this happens for the symmetric group on elements whose cycle decomposition has cycles of the same length, and also for compact unitary groups. We discuss some of the results due to me and others, and some framework to think about them for general reductive groups.

Faculty Talks

1 Omprokash Das

TITLE: Birational classification and the Minimal Model Program (MMP)

ABSTRACT: Two varieties X and Y are called birational if they have isomorphic Zariski open dense subsets, or equivalently, their function fields $K(X)$ and $K(Y)$ are isomorphic as algebras. Birationality defines an equivalence relation on the set of all varieties. The Minimal Model Program (MMP) or the Mori Program aims to find a ‘minimal’ object in each birational equivalence class. In this talk I will explain the main ideas of the Minimal Model Program.

2 Subhajit Goswami

TITLE: Geometry of random walk trajectories

ABSTRACT: Geometric properties of random walks display a rich phenomenology. To mention but a few examples, in planar setups one knows for instance that the outer boundary of a Brownian motion has Hausdorff dimension $4/3$ (originally conjectured by Mandelbrot and proved relatively recently in early 2000 following the discovery of Schramm–Loewner evolution (SLE)) and that several natural ‘observables’ (its occupation measure, thick points, uncovered set etc.) exhibit a (multi-)fractal structure. In the present talk, I will discuss some recent results related to the topology and geometry of random walk trajectories in higher dimensions which call for

the introduction of a novel random object called the “random interlacements” discovered in a seminal work by Sznitman (Ann. of Math. 2010). In the course of the talk, I will try to illustrate how some seemingly impenetrable questions about the random walk can be elegantly answered exploiting some deep connections with apparently unrelated areas of analysis and probability theory.

3 Radhika Gupta

TITLE: Tale of two geometries and two groups

ABSTRACT: I will introduce two notions of non-positively curved spaces - hyperbolic spaces and CAT(0) spaces, and look at properties of groups acting ‘nicely’ on such spaces. I will also introduce two groups - the mapping class group of a surface and the group of automorphisms of a finitely generated free group. These groups unfortunately are neither hyperbolic nor CAT(0). However, we will see some spaces on which these groups act ‘nicely enough’ and explore the geometry of these spaces.

4 Anand Sawant

TITLE: Smooth projective varieties over a field from the perspective of \mathbb{A}^1 -algebraic topology

ABSTRACT: In classical algebraic topology, the orientability of a compact, connected manifold can be characterized in terms of its top singular homology. I will describe the analogues of this for smooth projective varieties over an arbitrary perfect field when the singular homology is replaced with appropriate motivic homology theories. The talk will be based on joint work in progress with Fabien Morel.

Student Talks

5 Gaurav Aggarwal

TITLE: Joint equidistribution of approximates

ABSTRACT: The distribution of integer points on varieties has occupied mathematicians for centuries. In the 1950s Linnik used an “ergodic method” to prove the equidistribution of integer points on large spheres under a congruence condition. As shown by Maaß, this problem is closely related to modular forms. Subsequently, there were spectacular developments both from the analytic as well as ergodic side. I will discuss a more refined problem, namely the joint distribution of lattice points in conjunction with other arithmetic data. An example of such data is the “shape” of an associated lattice, or in number theoretic language, a Heegner point. In a completely different direction, a “Poincaré section” is a classical and useful tool in ergodic theory and dynamical systems. Recently, Shapira and Weiss constructed a Poincaré section for the geodesic flow on the moduli space of lattices to study joint equidistribution properties. Their work in fact is very general but crucially uses the fact that the acting group has rank one. In joint work with Anish Ghosh, we develop a new method to deal with actions of higher rank groups. I will explain this and, if time permits, some corollaries in Diophantine analysis.

6 Ritwik Chakraborty

TITLE: Ising models on hyperbolic groups

ABSTRACT: The Gibbs measure is a probability measure that describes a physical system with finitely many interacting components in equilibrium at a given temperature and is the starting point of Classical Statistical Mechanics. In the '60s, Dobrushin, Lanford, and Ruelle gave the first robust definition of Gibbs measures on infinite graphs. In probabilistic terms, these are probability measures that satisfy a certain spatial Markov property. I will talk about Gibbs measures for the Ising model on infinite graphs. It is known that at high temperatures there is a unique Gibbs measure. I will talk about ongoing work with Mahan Mj and Subhajit Goswami where we show that at low temperatures there are uncountably many mutually singular Gibbs measures for the Ising model on hyperbolic lattices of dimension $d \geq 2$ and more generally, on Cayley graphs of hyperbolic groups.

7 Subhadip Majumder

TITLE: Ramified class field theory of curves over local fields

ABSTRACT: In this talk we will discuss some results on class field theory of smooth projective curves over a local field with arbitrary ramification along a divisor. To deduce this we will introduce some new results on ramified duality and class field theory of 2-dimensional local fields. This is based on a joint work with Amalendu Krishna.

8 Biswajit Nag

TITLE: Relatively hyperbolic groups and drilling hyperbolic surface-bundles over graphs

ABSTRACT: Hyperbolic groups, introduced by Gromov, are generalisations of the fundamental groups of hyperbolic surfaces and closed compact hyperbolic 3-manifolds. Fundamental groups of closed hyperbolic 3-manifolds of finite volume satisfy a weaker notion of hyperbolicity: these groups are hyperbolic *relative* to their (possibly empty) collection of cusp subgroups. In this talk, we will describe examples of 3-dimensional complexes (not always manifolds) with relatively hyperbolic fundamental groups. These examples are constructed by first considering a hyperbolic surface-bundle over a graph, and then drilling (that is, removing a tubular neighbourhood of) an essential simple closed curve lying in a fibre.

This is joint work with Mahan Mj.

9 Ritankar Nath

TITLE: On the Atiyah Segal completion for equivariant algebraic K theory

ABSTRACT: The Atiyah Segal Completion theorem, proved jointly by Atiyah and Segal in 1969, establishes a relation between equivariant topological Bredon and Borel K theories. To be precise, for a compact Lie group G and a G -CW complex X , it establishes the isomorphism

$$\pi^* : K_G^*(X)_{I_G}^\wedge \rightarrow K^*((X \times EG)/G)$$

where $\pi : X \times EG \rightarrow X$ is the projection map where $K_G^*(X)_{I_G}^\wedge$ is the completion of Topological K theory group at the augmentation ideal I_G of the representation ring $R(G)$. In this talk, we will investigate the various successes and failures of the analogue of this theorem with equivariant Algebraic K theory groups. This talk is based on an ongoing joint work with Amalendu Krishna.

10 Niladri Patra

TITLE: Irreducibility results in moduli spaces of prime degree polynomials

ABSTRACT: In this talk, we will discuss a few irreducibility results in moduli spaces of odd prime degree polynomials with a marked critical point. To summarize them,

1) Families of conjugacy classes of polynomials for which the marked critical point is prefixed form irreducible quasi-affine subvarieties of the moduli space.

2) In the quasi-affine varieties mentioned above, the decreasing chain of subfamilies obtained by fixing the ramification index of the marked critical point, consists of irreducible quasi-affine subvarieties, except for the unicritical case.

3) In the unicritical case, these families are finite and each of them forms a single Galois orbit under the action of the absolute Galois group of \mathbb{Q} .

4) In the moduli space of cubic polynomials, conjugacy classes of polynomials for which the marked critical point is eventually 2-periodic are irreducible affine curves.

The results above are based on the articles [Pat1] and [Pat2].

References

[Pat1] Patra, Niladri. On irreducibility of prefixed algebraic sets in moduli spaces of prime degree polynomials, math.DS, 2023, arXiv:2305.04778v2

[Pat2] Patra, Niladri. Irreducibility of eventually 2-periodic curves in the moduli space of cubic polynomials. math.DS, 2023, arXiv:2305.19944v2

11 Sandeep S

TITLE: Kato complexes of reciprocity sheaves

ABSTRACT: To every homotopy invariant Nisnevich sheaf with transfers, we can associate a cycle complex by the work of Rost and Déglise. Kato associated a similar complex to Illusie's logarithmic de-Rham-Witt Sheaves over a field of positive characteristic p , which are not homotopy invariant.

Using the recent work of Binda, Rülling and Saito on the Gysin map, we obtain a generalisation of Kato's work to certain reciprocity sheaves. This is based on joint work with Anand Sawant.

12 Samiron Sadhukhan

TITLE: Brauer-Manin pairing and related duality theorems

ABSTRACT: In this talk, we'll discuss the Brauer–Manin pairing for curves over local fields. In characteristic zero, for a proper smooth curve, the perfectness of this pairing was proven by Lichtenbaum in the late '60s. In characteristic p , this result was proven by Shuji Saito in the '80s. The case of open curve over characteristic zero was settled by Scheiderer and van Hamel in 2003. We will discuss the case of open curves over characteristic p local field and in the process, describe a relevant duality theorem for cohomologies of logarithmic Hodge-Witt sheaves. This is joint work with A. Krishna and J. Rathore.

13 Soumyadip Sahu

TITLE: Reduction map in the higher K -theory of the rings of integers in number fields

ABSTRACT: The lecture aims to study the mod ℓ reduction maps in the higher K -theory of the ring of integers in a number field arising from the natural reduction maps at maximal ideals. We prove an explicit density estimate for the subset of finite places where the images of a fixed collection of elements vanish. We also provide a version of this result that applies to elements of different degrees. Our investigation unearths a simple connection between the reduction maps attached to the K -theory of the ring of integers and the stable cohomology of the general linear group that allows us to examine the image of Quillen's e -classes under reduction.

14 Balarka Sen

TITLE: Tight contact structures on hyperbolic homology 3-spheres

ABSTRACT: A contact structure on a 3-manifold is a nowhere integrable 2-plane distribution. Contact structures on 3-manifolds exhibit a dichotomy: tight vs. overtwisted. Eliashberg showed that overtwisted contact structures are classified purely by algebraic topology, and asked if all irreducible closed 3-manifolds admit tight contact structures. This remains an outstanding open problem in low-dimensional contact topology for the largest class of such 3-manifolds: hyperbolic rational homology 3-spheres. In this talk, I shall report on a joint work-in-progress with Mahan Mj, where we construct a large class of hyperbolic integer homology 3-spheres admitting arbitrarily large numbers of distinct tight contact structures, and study their behavior under geometric limits.

15 Saniya Wagh

TITLE: Character sheaves on tori over local fields

ABSTRACT: In this talk, I will explain recent work with Prof. Tanmay Deshpande. Let T be a torus over a complete discrete valuation field with an algebraically closed residue field k . Let L^+T be the connected commutative pro-algebraic group obtained by applying the Greenberg functor to the connected Néron model of T . We give a canonical isomorphism between the abelian group of multiplicative local systems (character sheaves) on L^+T and the inertial local Langlands parameters. We then relate this isomorphism to the classical Langlands correspondence for tori using the sheaf-function correspondence.