Tata Institute of Fundamental Research

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The Infosys Chandrasekharan Random Geometry Lecture Series

Professor Barak Weiss from Tel Aviv University will give a series of three lectures. Barak Weiss is a Professor of Mathematics at Tel Aviv University and an expert in homogeneous dynamics, dynamics on flat surfaces and on the geometry of numbers. He was an invited speaker at the 2022 ICM.

The Covering Volume of Lattices and Nearly Uniform Coverings

Abstract

Let L in $\mathbb{R}^n$ be a lattice and let $\mathcal{K}$ be a convex body. The covering volume of $\mathcal{K}$ w.r.t. L is the minimal volume of a dilate $r\mathcal{K}$ such that $L + r\mathcal{K} = \mathbb{R}^n$, normalized by the covolume of L. Pairs $(L, \mathcal{K})$ with small covering volume correspond to efficient coverings of space by copies of $\mathcal{K}$, translated by elements of L. Finding upper bounds on the covering volume as the dimension n goes to infinity, is a well-studied problem, with connections to practical issues arising in computer science and electrical engineering. In a recent paper with Ordentlich (EE, Hebrew U) and Regev (CS, NYU), we obtain substantial improvements to bounds of Rogers from the 1950???s. In another recent paper, we obtain bounds on the minimal volume of nearly uniform covers, where a pair $(L, \mathcal{K})$ gives an ϵ-nearly uniform cover if the ratio between $\max_x |\{l \in L : x \in l + \mathcal{K}\}|$ and $\min_y |\{l \in L : y \in l + \mathcal{K}\}|$ is at most $1 + \epsilon$. The key to these results are recent breakthroughs due to Dvir and others regarding the discrete Kakeya problem. I will give three lectures about these results, including history, applications, and some ideas of the proofs. No prerequisites beyond undergraduate material (measures, volumes, vector spaces over finite fields) will be assumed.

Time and Venue

Lecture 1: 10:00 am to 11:00 am October 18, AG–66

Lecture 2: 4:00 pm to 5:00 pm October 19, AG–69

Lecture 3: 4:00 pm to 5:00 pm October 21, AG–77