

Abstracts
(Listed in the order of the schedule)

Speaker: M. Ram Murty (Queen's Univ, Canada)

Title: Special values of Artin L -series.

Abstract: We will survey what is known and unknown about the transcendental nature of special values of Artin L -series. We also report on some recent joint work with Kumar Murty on the transcendence of special values of Hecke L -series and their relationship with Petersson inner products.

Speaker: H. Maier (Universitaet Ulm, Germany)

Title: The behaviour in short intervals of exponential sums over sifted integers (Joint work with A. Sankaranarayanan).

Abstract: We consider the Hardy-Littlewood approach to the Twin prime problem, which uses a certain exponential sum over prime numbers. We propose a conjecture on the behaviour of the exponential sum in short intervals of the argument. We first show that this conjecture implies the Twin prime conjecture. We then prove that an analogous conjecture is true for exponential sums over integers without small prime factors.

Speaker: A. Sankaranarayanan (TIFR, Mumbai)

**Title : Exponential sums over primes in residue classes
(Joint work with H. Maier).**

Abstract: We specialize a problem studied by Elliott, the behaviour of arbitrary sequences a_p of complex numbers on residue classes to prime moduli to the case $a_p = e(\alpha p)$. For these special cases, we obtain under certain additional conditions improvements on Elliott's results.

Speaker: S.D. Adhikari (HCRI: Allahabad)

Title: Visibility of Lattice points.

Abstract: If $d \geq 2$, we say that $\mathbf{a} \in \mathbb{Z}^d$ is *visible* from $\mathbf{b} \in \mathbb{Z}^d$ if there is no element of \mathbb{Z}^d on the straight line segment in-between \mathbf{a} and \mathbf{b} . One immediately deduces that (a, b) is visible from (c, d) if and only if $\gcd(c-a, d-b) = 1$, and, more generally, that \mathbf{a} is visible from \mathbf{b} if and only if the gcd of the coordinates of $(\mathbf{a} - \mathbf{b})$ equals 1. We say that $A \subset \mathbb{Z}^d$ is visible from $B \subset \mathbb{Z}^d$ if, for each point $\mathbf{a} \in A$ there is some $\mathbf{b} \in B$ such that \mathbf{a} is visible from \mathbf{b} . After an introductory discussion on some of the early results, we take up a particular problem in this area, namely the problem of determining the size of the smallest $B \subset \mathbb{Z}^d$ such that the set of integer lattice points inside a rectangular box A is visible from B . This is one of the list of problems compiled by (L. & W.) Moser and also appears in 'Research Problems in Discrete Geometry', Springer, 2005, by Peter Brass, William Moser and János Pach and in 'Lattice Points', John Wiley and Sons, New York, 1989, by Erdős, Gruber and Hammer. We shall discuss on some recent result obtained in collaboration with Andrew Granville after describing some earlier results obtained jointly with R. Balasubramanian and Yong-Gao Chen.

Speaker: G. Wuesthloz (ETH, Switzerland)
Title : Newton, Leibnitz and transcendence.

Abstract: In LEMMA XXVIII of the Principia Newton gives a statement about the transcendence of an area function related to the movement of planets. The Lemma turned out to become a starting point of a long discussion about whether the proof was correct. Huygens gave as a counterexample the Bernoullian lemniscate. In this context Leibniz made a conjecture on the transcendence of values of rational integrals on curves taken between two algebraic points on the curve, as one would formulate it in modern terms. We give a complete answer to this conjecture and it turns out that such an integral can be algebraic in some cases and is transcendental in other. Our work implies most of the classical transcendence results.

Speaker: S. Boecherer, (Univ. of Mannheim, Germany)
Title: On a formula of B.Heim.

Abstract: By comparing several restrictions of Siegel Eisenstein series of degree 3 B.Heim found a remarkable (and somewhat mysterious) formula relating central values of triple L -functions to other critical values. It is quite common that such formulas can be generalized by using equivariant differential operators. However, there are no differential operators equivariant under all the restrictions involved. We show how one can overcome this difficulty (joint work with B.Heim).

Speaker: A. Perelli (Dipartimento di Matematica, Italy)
Title: Non-linear twists of L -functions, I.

Abstract: Non-linear twist of an L -function $L(s)$ is a Dirichlet series obtained by twisting the coefficients of $L(s)$ by a non-linear exponential. In this talk we will survey joint work with J.Kaczorowski on the analytic properties of certain non-linear twists of the L -functions of the Selberg class, as well as several applications of such twists.

Speaker: J. Kaczorowski (Adam Mickiewicz Univ, Poland)
Title: Non-linear twists of L -functions, II.

Abstract: A non-linear twist of an L -function $L(s)$ is a Dirichlet series obtained by twisting the coefficients of $L(s)$ by a non-linear exponential. In this talk we will survey joint work with A. Perelli on the analytic properties of certain non-linear twists of the L -functions of the Selberg class, as well as several applications of such twists.

Speaker: K. Srinivas (IMSc, Chennai)
Title: Hardy's theorem for General Dirichlet Series.

Abstract: Hardy was the first to show that there are infinitely many zeros of the Riemann zeta function on the critical line. We shall discuss this result for the degree 2 functions in the Selberg class.

Speaker: J. Andersson (Uppsala University, Sweden)
Title: On a problem of Ramachandra and the Riemann zeta function.

Abstract: Let $\Phi(n)$ be an increasing function. We answer the question of whether there exists a $H > 0$ such that

$$\lim_{N \rightarrow \infty} \min_{|a_n| \leq \Phi(n)} \int_0^H \left| 1 + \sum_{n=2}^N a_n n^{it-1} \right|^2 dt > 0,$$

in terms of the growth of the function $\Phi(n)$. This tells us rather precisely when a conjecture of Ramachandra is true or false. As an application of this result, we give a lower bound for the mean square of the Riemann zeta function in short intervals close to the line $\text{Re}(s) = 1$.

Speaker: Haseo Ki (Yonsei University, Republic Of Korea)
Title: On the zeros of Weng's zeta functions.

Abstract: L. Weng introduced new zeta functions based on Eisenstein periods. We prove that all zeros of Weng's zeta functions for $SL(2, 3, 4, 5)$, $Sp(4)$ and G_2 are simple and on the critical line.

Speaker: K. Tsang (University of Hong Kong, Hong Kong)
Title: An Extension of the Brun-Titchmarsh Inequality.

Abstract: The celebrated Brun-Titchmarsh inequality

$$\pi(x; k, a) = \sum_{\substack{p \leq x \\ p \equiv a \pmod{k}}} 1 \ll \frac{x}{\varphi(k) \log \frac{x}{k}},$$

which holds uniformly in $k < x$ is a useful supplement to the Siegel-Walfisz theorem, and the Bombieri-Vinogradov theorem. This talk concerns an extension of the Brun-Titchmarsh inequality to numbers with given number of prime factors. This is a joint work with T.H. Chan and S.K.K. Choi.

Speaker: R. Schulze-Pillot (Universität des Saarlandes, Germany)
Title: Representations of quadratic forms by quadratic forms.

Abstract: We discuss the problem of determining which positive definite integral symmetric $n \times n$ - matrices T are represented by a given positive definite integral symmetric $m \times m$ - matrix A in the form $T = {}^tXAX$ with integral X . This problem has been treated in the past by arithmetic and by analytic methods (Hardy Littlewood method and modular forms theory) with the goal to show under suitable restrictions on m and n that all T of sufficiently large minimum, which are represented locally everywhere, are represented globally. Recently enormous progress has been made by methods from ergodic theory in the work of Jordan and Ellenberg. We compare the various approaches and show how previous results of Kitaoka can be combined with the new method to obtain results about representation of binary forms by forms of rank 6 and some similar cases. We also discuss some questions about Jacobi forms which arise when one tries to replace the condition on the minimum of T by other growth conditions.

Speaker: J.M. Deshouillers (Universités Bordeaux, France)
Title: On the distribution modulo one of the mean values of some arithmetical function.

Abstract: I shall present the positive answer, given by Henryk Iwaniec and myself, to a question raised by Florian Luca: is the sequence consisting of the arithmetic (resp. geometric) mean values of the Euler φ function uniformly distributed modulo 1?

Speaker: J. Steuding (University of Wuerzburg, Germany)
Title: New results on the value-distribution of the Riemann zeta-function on the critical line.

Abstract: We present joint work with Justas Kalpokas (Vilnius). We investigate the intersections of the curve $t \mapsto \zeta(\frac{1}{2} + it)$ (for real t) with the real axis. We show that if Riemann's hypothesis is true, the mean-value of those real values exists and is equal to 1. Moreover, we show unconditionally that the zeta-function takes arbitrarily large real values on the critical line.

Speaker: Ravi Raghunathan (IIT, Mumbai)
Title: On the algebraic independence of cuspidal automorphic L -functions.

Abstract: In this talk we prove the algebraic independence of cuspidal automorphic L -functions of GL_n over the rational numbers. Using automorphic induction, the results extend to solvable extensions of the rationals and conjecturally to any number field.

Speaker: J. Hoffstein (Brown University, USA)
Title: Multiple Dirichlet series and Shifted Convolutions.

Abstract: Let ℓ_1, ℓ_2 be two positive integers and let f and g be cusp forms of even weight k for $\Gamma_0(N_0)$, having Fourier expansions

$$f(z) = \sum_{m \geq 1} a(m)e^{2\pi imz} \quad g(z) = \sum_{m \geq 1} b(m)e^{2\pi imz}.$$

I'll define a certain Dirichlet series and a related double Dirichlet series and show how the double Dirichlet series can be meromorphically continued to \mathbf{C}^2 and how its analytic properties can be used to discover properties of averages of shifted sums related to f and g . The Dirichlet series is

$$D(s; h) = \sum_{m_2 \geq 1, m_1 \ell_1 = m_2 \ell_2 + h} \frac{a(m_1)\bar{b}(m_2)}{(m_2 \ell_2)^{s+k-1}}$$

and the double Dirichlet series is

$$Z(s, w) = \sum_{h \geq 1} \frac{D(s; h)}{h^{w+(k-1)/2}}.$$

The related average shifted sums are of the form

$$S(x; y) = (\ell_1 \ell_2)^{(k-1)/2} \sum_{m_1 \ell_1 = \ell_2 m_2 + h, m_2 \sim x, h \sim y} \frac{a(m_1)\bar{b}(m_2)}{(\ell_2 m_2)^{k-1}}.$$

Speaker: B. Ramakrishnan (HCRI, Allahabad)
Title: A canonical subspace of modular forms of half-integral weight.

Abstract: In this talk, we describe a canonical subspace of modular forms of weight $k + 1/2$ on $\Gamma_0(4N)$, where N is odd and square free. This subspace has the property similar to the Kohnen's plus space in the sense that under a class of Shimura maps, this subspace is mapped to modular forms of weight $2k$ and level N . As an application, we obtain a formula for $r_{2k+1}(|t|n^2)$, the number of ways of representing $|t|n^2$ as a sum of $2k + 1$ integer squares, for all $n \geq 1$, where t varies over a class of square free integers. This is a joint work with Sanoli Gun and M. Manickam.

Speaker: Y.J. Choie (POSTECH, Republic of Korea)
Title: Quasimodular forms.

Abstract: We study a various connection and applications among Quasimodular forms, Jacobi like forms and Pseudodifferential Operators.

Speaker: M. Waldschmidt (Universite Paris, France)
Title: Criteria for: irrationality, linear independence, transcendence, algebraic independence.

Abstract: Most irrationality proofs rest on the following criterion: a real number x is irrational if and only if, for any $\epsilon > 0$, there exists two rational integers p and q with $q > 0$, such that

$$0 < |qx - p| < \epsilon.$$

We survey generalisations of this criterion to linear independence, transcendence and algebraic independence.

Speaker: T.D. Browning (Univ. of Bristol, UK)
Title: Least zero of a cubic form.

Abstract: A famous result of Cassels provides a search bound for the least non-trivial integer solution of the equation $Q = 0$, if it exists, for any integral non-singular quadratic form Q . In this talk I will discuss the analogous situation for cubic forms, the treatment of which requires better information about the density of zeros of cubic forms modulo a prime p . This is joint work with Rainer Dietmann and Peter Elliott.

Speaker: Y. Bugeaud (Univ. Louis Pasteur, France)
Title: Diophantine approximation and Cantor sets.

Abstract: We survey various results asserting the existence, on the triadic Cantor set (denoted by C), of real numbers with prescribed Diophantine properties. In particular, we explain how the theory of continued fractions allows one to construct easily elements of C having any given irrationality exponent. However, numerous questions remain unanswered.

Speaker: W. Kohnen (Universitaet Heidelberg, Germany)
Title : Sign changes of Fourier coefficients of cusp forms of half-integral weight.

Speaker: A. Mukhopadhyay (IMSc, Chennai)
Title: Class numbers with many prime factors.

Abstract: We study the question of counting the number of quadratic fields whose class number is divisible by a given prime. Results of this well studied question can be used to produce infinite family of quadratic fields with ‘many’ prime factors. Finally we describe a family of number fields of a given degree d whose class number has ‘many’ prime factors. This is a joint work with K. Chakraborty and F. Luca.

Speaker: F. Luca (Instituto de Matematicas, Mexico)
Title: Counting dihedral and quaternionic extensions.

Abstract: We give asymptotic formulas on the numbers of biquadratic extensions of \mathbf{Q} that admit a quadratic extension which is a Galois extension of \mathbf{Q} with a prescribed Galois group, for example, with a Galois group isomorphic to the Quaternionic group. Our approach is based on a combination of the theory of systems of quadratic equations with some analytic tool such as the Siegel–Walfisz Theorem and double oscillations theorems.

The results presented in this talk have been obtained in joint work with Fouvry, Pappalardi and Shparlinski.

Speaker: B. Sury (ISI, Bangalore)

Title: Density results for primes in progressions dividing certain sequences.

Abstract: In 1641, Fermat conjectured among other things that if a prime p divides $3^n + 1$ for some n , then $p \not\equiv 1 \pmod{12}$. Similarly, he conjectured that if a prime p divides $5^n + 1$ for some n , then $p \not\equiv 1 \pmod{10}$. Later, these have been refuted by Schinzel and others. Our main theorem (proved in collaboration with P. Moree) implies that there is even a positive Dirichlet density of primes for which the conclusions of these conjectures are false. For instance, the densities of primes refuting these conjectures are respectively, $1/6$ and $1/12$. In general, for positive integers a, b, c, d with $(c, d) = 1$ and $a \not\equiv b \pmod{d}$, we show that the set of primes congruent to $c \pmod{d}$ which divide $a^n + b^n$ for some n has a Dirichlet density $\delta_{a,b}(c, d)$ which turns out to be a rational number; we compute it explicitly. In case $\delta_{a,b}(c, d) = 0$, we use elementary arguments like quadratic reciprocity to show that there are at most finitely many primes p with this property. Likewise if $\delta_{a,b}(c, d) = 1/\varphi(d)$, using quadratic reciprocity, one can show that in each case there are at most finitely many exceptions. We also deduce density results for primes dividing a member of the sequence $a^n + (-b)^n$ in any progression $c \pmod{d}$.

Speaker: O. Ramare (Universite de Lille, France)

Title: Towards a geometrical setting for the weighted sieve.

Abstract: We mix an early approach to the weighted sieve due to Bombieri with more geometrical ideas to guess a better choice for the majorizing sequence. This leads to better bounds when the dimension is large. We prove for instance that there exist infinitely many 8-uples of integers, such that the product of the integers composing each 8-uple has not more than 28 prime factors. We get further examples by replacing the couple (8,28) by (10,37), (11,42), (15,60) or by (16,65). This work is still in progress.

Speaker: T.N. Shorey (TIFR, Mumbai)

Title: Divisibility properties of Laguerre polynomials.