

**Discussion Meeting on Analytic Number Theory 2015
(DMANT2015)
Titles and Abstracts**

**Speaker: M. Ram Murty (Queen's University, Canada)
Title : The Chowla and Erdős problems**

Abstract : We will present some recent work on the Chowla and Erdős problems regarding non-vanishing of Dirichlet series at $s=1$, attached to periodic functions.

**Speaker : Hajdu Lajos (University of Debrecen, Hungary)
Title : The Jacobsthal function and its applications
(joint work with N. Saradha and R. Tijdeman)**

Abstract : The Jacobsthal function $g(n)$ is defined in the following way. For $n \geq 1$ let $g(n)$ denote the smallest m such that in any block of m consecutive integers one can find an integer k with $\gcd(n, k) = 1$. In the frame of the talk we give a survey of some results concerning $g(n)$ and its applications. As a motivation, by the help of a nice theorem of Pomerance we illustrate the importance of $g(n)$ for bounding the smallest primes in arithmetic progressions; we present a disproof of a conjecture of Jacobsthal from 1962 (stating that certain extremal values of $g(n)$ should be linked to values of n obtained as the product of the first r primes); we present a proof of a conjecture of Recaman from 1978 (stating that the only prime p for which the first p primes form a full residue system modulo p is $p = 2$); we present a proof for a more general conjecture of Pomerance from 1980. In case of the last problem the complete solution is due to Togbé and Yang (2014).

**Speaker : Ravi Raghunathan (Indian Institute of Technology, Mumbai)
Title : On the poles of character twists of L -functions**

Abstract : We use a method of A. Booker to investigate the poles of character twists of Dirichlet series of degree 2. Our results can be applied towards primitivity results in the Selberg class.

**Speaker : S. Adhikari (Harish-Chandra Research Institute, Allahabad)
Title : Zero-sum theorems: Some algebraic methods**

Abstract : Let G be a finite abelian group, written additively. Zero-sum theorems in Combinatorial Number Theory are mainly concerned with the minimum length of an arbitrary sequence over G guaranteeing the existence of a subsequence T , often with some required length, such that the elements of T sum to 0, where 0 is the identity element of the group G .

Among the various methods that have been used to attack these problems, here we discuss some algebraic methods used to deal with the classical questions as well as for some weighted generalizations of these problems.

Speaker : Purusottam Rath (Chennai Mathematical Institute, Chennai)

**Title : Transcendental sums related to the zeros of Riemann zeta function
(Joint work with Sanoli Gun and Ram Murty)**

Abstract : We study the transcendental nature of functions related to the zeros of the Riemann Zeta function specialized at algebraic points. These functions are potential source of “new” transcendental numbers.

Speaker : K. Srinivas (Institute of Mathematical Sciences, Chennai)

**Title : On the angles of eigen forms
(This is a joint work in progress with M. Ram Murty)**

Abstract : In this talk we shall discuss the problem of counting the number of pairs of eigen forms (f, g) of weight k and level N such that $a_p(f) = a_p(g)$ where $a_p(f)$ denotes the p -th Fourier coefficient of f . Here p is a fixed prime.

Speaker : Winfried Kohnen (University of Heidelberg, Germany)

**Title : Characterization of cusp forms by means of the growth of their
Fourier coefficients**

Abstract : In this talk, we characterize cusp forms by means of the growth of their Fourier coefficients.

Speaker : Abhishek Saha (University of Bristol, UK)

Title : Sup norms of cusp forms in the level aspect

Abstract : This will be a talk about the problem of estimating the ratio of the L^∞ and L^2 norms of newforms on $GL(2)$ as the conductor (level) of the forms vary. I will especially focus on the powerful level aspect; this is when high powers of primes divide the level. After some survey remarks, I will describe two recent results of mine. One of them is a *lower bound* for the ratio that is “large” whenever the central character is sufficiently ramified with respect to the level. This result extends some previous work of N. Templier, and being purely local, applies to general number fields. The other is an *upper bound* for the L^∞ norm in the classical case (where the base field is \mathbb{Q}) that improves upon the trivial bound. This extends previous work of Blomer-Holowinsky, Harcos-Templier, and others, who dealt with the case of increasing square-free levels. Time permitting, I will end with some remarks about how one might be able to drastically improve the second result in the powerful aspect.

**Speaker : R. Balasubramanian (Institute of Mathematical Sciences,
Chennai)**

Title : To Be Announced

Abstract : To Be Announced

Speaker : Jeffrey Hoffstein (Brown University, USA)

Title : Second moment formulas for Rankin-Selberg convolutions and character twists of $GL(2)$ L -series

Abstract : I'll describe recent joint work with Min Lee in which we apply the meromorphic properties of shifted multiple Dirichlet series to certain second moment problems. One application is to second moments of Rankin-Selberg convolutions averaged over the spectrum, essentially a generalization of some fourth moment estimates of Ivic and Motohashi. Another application is to the problem of expressing the mean square of $GL(2)$ L -series averaged over characters as a main term plus an error term.

Speaker : Saurabh Kumar Singh (TIFR, Mumbai)

**Title : On the Riesz means of $\frac{n}{\phi(n)}$
(joint work with A. Sankaranarayanan)**

Abstract : Let $\phi(n)$ denote the Euler-totient function. We study the error term of the general k -th Riesz mean of the arithmetical function $\frac{n}{\phi(n)}$ for any positive integer $k \geq 1$, namely the error term $E_k(x)$ where

$$\frac{1}{k!} \sum_{n \leq x} \frac{n}{\phi(n)} \left(1 - \frac{n}{x}\right)^k = M_k(x) + E_k(x).$$

For instance, the upper bound for $|E_k(x)|$ established here improves the earlier known upper bounds for all integers k satisfying $k \gg (\log x)^{1+\epsilon}$.

Speaker : Kaneenika Sinha (IISER, Pune)

**Title : Average ranks of certain abelian varieties
(joint work with H. Iwaniec)**

Abstract : In 1995, A. Brumer and R. Murty, independently investigated the behaviour of ranks of Jacobians of modular curves. Conditional upon the Riemann Hypothesis for L -functions of newforms, they predicted the behaviour of these ranks "on average." In a later work, Michel, Kowalski and VanderKam exploited a very beautiful technique in analytic number theory, namely the mollification technique, to estimate the average ranks unconditionally. We survey their results and techniques. In joint work with H. Iwaniec, we also show how the mollification method helps us to study the non-vanishing of central values of the Rankin Selberg L -functions.

Speaker : Holowinsky, Roman (Ohio-State University, USA)

**Title : Sup-norms of Hecke-Maass cusp forms
(joint work with G. Ricotta and E. Royer)**

Abstract : We shall highlight recent advancements in the study of sup-norms for Hecke-Maass cusp forms on higher rank groups. In particular, this talk will provide a summary of joint work with G. Ricotta and E. Royer regarding an explicit non-trivial bound in the eigenvalue aspect for the sup-norm of an $SL_3(\mathbb{Z})$ Hecke-Maass cusp form restricted to a compact set.

Speaker : Yann Bugeaud (University of Strasbourg, France)

Title : On the continued fraction expansion of algebraic numbers

Abstract : We survey recent results on the continued fraction expansion of algebraic numbers of degree at least three. We show that, if the sequence of partial quotients of a real number enjoys certain combinatorial properties, then this number must be transcendental. As a consequence, the infinite word composed of the partial quotients of an algebraic number of degree at least three cannot have sublinear block complexity and it cannot be generated by a finite automaton.

Speaker : Dinesh Thakur (University of Rochester, USA)

Title : Algebraic relations between zeta and multizeta values

Abstract : We will explain what is known and conjectured about this topic in the number field and function field case, and how the topic relates to interesting mathematical structures.

Speaker : Shanta Laishram (Indian Statistical Institute, Delhi)

**Title : Fibonacci numbers of the form $x^a \pm x^b \pm 1$
(joint work with F. Luca)**

Abstract : Fibonacci numbers $(F_n)_{n \geq 0}$ given by $F_0 = 0, F_1 = 1$ and $F_{n+2} = F_n + F_{n+1}$ for $n \geq 0$ is the most well known and widely studied recurrence sequence. A recent breakthrough result of Bugeaud, Mignotte and Siksek states that Fibonacci numbers are perfect powers only for $F_0 = 0, F_1 = 1, F_2 = 1, F_6 = 8$ and $F_{12} = 144$. In this talk, we show that a related Diophantine equation $F_n = x^a \pm x^b \pm 1$ has only finitely many positive integer solutions (n, x, a, b) with $\max\{a, b\} \geq 2$ and x having exactly two distinct prime factors.

Speaker : Helmut Maier (University of Ulm, Germany)

**Title : Asymptotics for high moments of Brjuno's function
(joint work with Michael Rassias (University of Princeton))**

Abstract : We give an overview on new results from joint work of the speaker with Michael Rassias (University of Princeton) and related results from work of other authors. These results are related to the distribution of cotangent sums.

Speaker : Guangshi Lü (Shandong University, Jinan, China)

Title : On Fourier coefficients of automorphic forms

Abstract : Fourier coefficients of automorphic forms are interesting and important objects in modern number theory. In this talk, the speaker will introduce some recent progress on some problems related to Fourier coefficients of automorphic forms. In particular, we shall talk about some problems related to Fourier coefficients of cusp forms and theta series.

Speaker : B. Sury (Indian Statistical Institute, Bengaluru)

Title : l -Class groups of cyclic extensions of prime degree $\frac{l}{Q(\zeta_l)}$

(joint work with Dipramit Majumdar and Manisha Kulkarni)

Abstract : Let K/F be a cyclic extension of number fields with degree a prime l . If F has class number coprime to l , the structure of the l -Sylow subgroup of the class group of K is studied. In particular, when F contains the l -th roots of unity, the rank of the l -Sylow subgroup of K is obtained using genus theory. Our work is mainly influenced by that of Frank Gerth III who studied the case of the prime 3. Generalizing his approach, some results for general l are obtained. Following that, more complete results for the prime 5 are obtained when F is the cyclotomic field of degree 4. The rank of the 5-class group of K is expressed in terms of power residue symbols. We compare our results with tables obtained using SAGE (the latter is under GRH).

Speaker : Kohji Matsumoto (Nagoya University, Japan)

Title : Desingularization of multiple zeta-functions

(joint work with H. Furusho, Y. Komori and H. Tsumura.)

Abstract : We introduce the method of desingularization of multiple zeta-functions of generalized Euler-Zagier type, under the motivation of finding suitable rigorous meaning of the values of multiple zeta-functions at non-positive integer points. The desingularized multiple zeta-function turns to be entire, and is written by a suitable finite linear combination of usual multiple zeta-functions. It is shown that specific combinations of Bernoulli numbers attain special values of desingularized zeta-function at non-positive integer points.

Speaker : Sanoli Gun (Institute of Mathematical Sciences, Chennai)

Title : Many faces of Euler's constant

(joint work with Tapas Chatterjee, Ekata Saha and Sneh Bala Sinha)

Abstract : In this talk, we discuss the transcendental nature of various manifestations of Euler's constant.

Speaker : B. Ramakrishnan (Harish-Chandra Research Institute, Allahabad)

Title : Newforms of half-integral weight

(joint work with M. Manickam and Jaban Meher)

Abstract : we present our recent work on the theory of newforms of half-integral weight of levels $8N$ and $16N$, where N is odd and square-free.

Speaker : Chaohua Jia (Institute of Mathematics, Academia Sinica, Beijing, China)

Title : Kloosterman sums with multiplicative coefficients

Abstract : In this lecture, we shall talk about the Kloosterman sums, its history, progress and some applications. We shall introduce a new result on the Kloosterman sums with multiplicative coefficients.

Speaker : Divyum Sharma (TIFR, Mumbai)
Title : Number of solutions of Thue inequalities
(joint work with N. Saradha)

Abstract : Let $F(X, Y) \in \mathbb{Z}[X, Y]$ be a form of degree $r \geq 3$, irreducible over \mathbb{Q} and having at most $s + 1$ non-zero coefficients. Let h be a non-zero integer. Siegel proposed that the number $N_F(h)$ of integer solutions of the Thue inequality

$$|F(X, Y)| \leq h$$

may be bounded only in terms of s and h . Mueller and Schmidt showed that

$$N_F(h) \ll s^2 h^{2/r} (1 + \log h^{1/r}).$$

Further, they conjectured that s^2 may be replaced by s . In this talk, we present some recent contributions in this direction.

Speaker : R. Thangadurai (Harish-Chandra Research Institute, Allahabad)

Title : On Prime k -tuple Conjecture
(joint work with G. Kasi Viswanadham)

Abstract : We shall discuss the relation between the well-known Prime k -tuple Conjecture and another conjecture on the minimal length of interval containing precisely k prime numbers.

Speaker : T.N. Shorey (Indian Institute of Technology, Mumbai)

Title : The product of two or more factorials being a factorial
(joint work with Saranya G. Nair)

Abstract : Under explicit abc conjecture, we confirm a difficult and long standing conjecture of Hickerson if $n!$ is a product of two or more factorials then $n \leq 16$.