“Discussion meeting on Probability theory”

October 6 - 10, 2025
Tata Institute of Fundamental Research, Mumbai

LECTURE SCHEDULE

Lecture Timings	Monday (6 October)	Tueday (7 October)	Wednesday (8 October)	Thursday (9 October)	Friday (10 October)
09:30 – 10:30	–	Ridhipratim Basu (AG-66)	Krishanu Maulik (AG-66)	Arijit Chakrabarty (AG-66)	Tridib Sadhu (AG-66)
10:30 – 11:15	TEA BREAK				
11:15 – 12:15	Hariharan Narayanan (AG-66)	Sudeshna Bhattacharjee (AG-66)	Moumanti Podder (AG-66)	–	Parthanil Roy (AG-66)
12:15 – 14:15	LUNCH				
14:15 – 15:15	Yogeshwaran D (AG-80)	Atul Shekhar (AG-66)	Anirban Basak (AG-66)	Soumendu S. Mukherjee (AG-66)	Conference end
15:15 – 16:00	TEA BREAK				
16:00 – 17:00	Ayan Bhattacharya (AG-66)	Free afternoon	Open problem session (AG-69)	Siva Athreya (AG-66)	–
17:30 – 18:30	Manjunath Krishnapur (AG-66)			–	–

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Abstracts of Talks

School of Mathematics
Tata Institute of Fundamental Research

Title of Talks

Hariharan Narayanan	*Sums of GUE matrices and concentration of hives from correlation decay of eigengaps*
Yogeshwaran D	*Invariant matchings of random points and hyperuniformity*
Ayan Bhattacharya	*Heavy-Tailed Branching Random Walk: Past, Present, and Future*
Manjunath Krishnapur	*Minimum possible area of a polynomial lemniscate*
Ridhipratim Basu	*First passage percolation on hyperbolic groups*
Sudeshna Bhattacharjee	*Characterization of global solutions to the KPZ fixed point with the same slope*
Atul Shekhar	*Asymptotics of expected number of components for random lemniscate with iid roots*
Krishanu Maulik	*Asymmetrically Step Reinforced Random Walk with Regularly Varying Memory*
Moumanti Podder	*A model of market economics inspired by random walks with long memory*
Anirban Basak	*Large deviations in random graphs*
Arijit Chakrabarty	*Top eigenvalues and eigenvectors of inhomogeneous Erdős-Rényi random graphs*
Soumendu S. Mukherjee	*Elephants on Trees: A Non-Markovian Random Walk on Finitely Generated Non-Abelian Infinite Groups*
Siva Athreya	*Interplay of vertex and edge dynamics for dense random graphs*
Tridib Sadhu	*How Singularities in Large Deviations Emerge in Stochastic Processes*
Parthanil Roy	*Phase Transitions for Elephant Random Walks with Two memory Channels*

Abstracts

Monday, 6 October 2025 (11:15-12:15)

Speaker : Hariharan Narayanan
Title : Sums of GUE matrices and concentration of hives from correlation decay of eigengaps

Associated to two given sequences of eigenvalues is a natural polytope, the polytope of augmented hives with the specified boundary data, which is associated to sums of random Hermitian matrices with these eigenvalues. As a first step towards the asymptotic analysis of random hives, we show that if the eigenvalues are drawn from the GUE ensemble, then the associated augmented hives exhibit concentration as the number of eigenvalues tends to infinity.Our main ingredients include a representation due to Speyer of augmented hives involving a supremum of linear functions applied to a product of Gelfand–Tsetlin polytopes; known results by Klartag on the KLS conjecture in order to handle the aforementioned supremum; covariance bounds of Cipolloni–Erdös–Schröder of eigenvalue gaps of GUE; and the use of the theory of determinantal processes to analyze the GUE minor process. This is a joint work with Scott Sheffield and Terence Tao.

Monday, 6 October 2025 (14:15-15:15)

Speaker : Yogeshwaran D
Title : Invariant matchings of random points and hyperuniformity

This talk investigates will focus on the asymptotic variances of random point clouds under invariant matchings. We establish natural mixing criteria under which invariant matchings preserve the asymptotic variance of these point configurations. Special emphasis is placed on the phenomenon of hyperuniformity, characterized by vanishing asymptotic variance, which reflects long-range order and regularity in point clouds. By constructing suitable matchings, we demonstrate methods to verify or refute hyperuniformity in various random configurations. In particular, we introduce a "hyperuniformerer" construction that transforms any ergodic point cloud into a hyperuniform configuration by randomizing each point within its cell of a fair spatial partition. Additionally, we explore the question of whether any hyperuniform configuration can be well matched to a lattice, representing the simplest example of a hyperuniform point cloud.

Monday, 6 October 2025 (16:00-17:00)

Speaker : Ayan Bhattacharya
Title : Heavy-Tailed Branching Random Walk: Past, Present, and Future

In this talk, we focus on branching random walks with heavy-tailed displacements and investigate the asymptotic behavior of the extreme positions when the underlying genealogical tree is given by a supercritical Bienaymé–Galton–Watson process. We will also discuss upper large deviations of the associated point process. In addition, some variants of the model will be considered, highlighting both the established results and the key open questions. This talk is based on joint work with Nina Gantert, Piotr Dyszewski, and Zbigniew Palmowski.

Monday, 6 October 2025 (17:30-18:30)

Speaker : Manjunath Krishnapur
Title : Minimum possible area of a polynomial lemniscate

Erdős-Herzog-Piranian (1958) raised many questions about metric properties of polynomial lemniscates. One of these was the problem of finding the minimum possible area of $\{z \in \mathbb{C} : |p(z)| < 1\}$ over all monic polynomials p of degree n with all zeros inside the closed unit disk. The best known results on the minimum area are the lower bound $\frac{1}{n^4}$ (Pommerenke, 1961), and the upper bound $\frac{1}{(\log\log n)^{0.5-u}}$ for any $u > 0$ (Wagner, 1988). In joint work with Erik Lundberg, Fedor Nazarov, and Koushik Ramachandran, we show that the minimum area is of order $\frac{1}{\log n}$. For lemniscates of other levels $\{z \in \mathbb{C} : |p(z)| < t\}$, we show that the minimal area is of order $\frac{1}{n}$ if $t < 1$ and of order $\frac{1}{\log\log n}$ if $t > 1$.

Tuesday, 7 October 2025 (09:30-10:30)

Speaker : Ridhipratim Basu
Title : First passage percolation on hyperbolic groups

I shall discuss first passage percolation on Cayley graphs of Gromov hyperbolic groups under mild conditions on the passage time distribution. Appealing to deep geometric and topological facts about hyperbolic groups and their boundaries, several questions become more tractable in this set up compared to their counterparts in Euclidean lattices. In particular, i shall describe several results about time constants, fluctuations, coalescence of geodesics, and exceptional directions where coalescence fails. Some of the results are parallel to what are expected in Euclidean background geometry, while substantially different features are exhibited in other aspects.

Tuesday, 7 October 2025 (11:15-12:15)

Speaker : Sudeshna Bhattacharjee
Title : Characterization of global solutions to the KPZ fixed point with the same slope

The KPZ fixed point is a Markov process on the space of upper semi-continuous functions. It is the conjectured universal scaling limit of height functions for models of the KPZ universality class and has been shown to be such for many solvable (and even some unsolvable) models. The directed landscape provides a coupling for the growth of the KPZ fixed point from all initial conditions. Under this coupling, starting from an initial condition, the KPZ fixed point evolves forward in time as a solution of a variational formula. A key feature of this evolution is that the asymptotic slope is conserved. For a fixed realization of the directed landscape, it is an interesting question to characterize all global solutions (i.e., defined for both forward and backward times) with a given slope. It is known that, with probability one, there exists a random set of exceptional slopes. For slopes outside this set, the global solution with the prescribed slope is unique, whereas for exceptional slopes there are at least two such solutions. In this talk we give a full characterization of the global solutions with these exceptional slopes. In particular, we show that each exceptional slope admits uncountably many global solutions. The talk is based on a joint work with Ofer Busani and Evan Sorensen.

Tuesday, 7 October 2025 (14:15-15:15)

Speaker : Atul Shekhar
Title : Asymptotics of expected number of components for random lemniscate with iid roots

For a random polynomial with iid roots in a disc, we consider its level sets (aka lemniscates). We obtain asymptotics for the expected number of connected components. This number shows a distinct behaviour at the critical radius $R = 1$. This can be accredited to a pairing phenomenon between roots and critical points of the given polynomial. This talk is based on a joint work with Koushik Ramachandran and Subhajit Ghosh.

Wednesday, 8 October 2025 (09:30-10:30)

Speaker : Krishanu Maulik
Title : Asymmetrically Step Reinforced Random Walk with Regularly Varying Memory

Schutz and Trimper (2004) introduced Elephant Random Walk (ERW) as a variant of simple symmetric random walk with memory. In ERW, a step is chosen at random from the entire past history of the walk and either, with probability p, the selected step is repeated or a fresh independent step is taken with the complementary property. Here p is called the memory parameter. This has also been alternatively called Step Reinforced Random Walk. The walk has drawn wide attention over the last two decades since its introduction, as it shows phase transition based on p and anomalous behaviour. Laulin (2022) proposed a variant where the past step is selected with probability proportional to a weight sequence which is regularly varying of index $\gamma > -1$. The law of large numbers and fluctuations around the large number limit has been obtained for certain values of γ and p. We provide a complete analysis and functional limit theorems of the process for all values of γ and p and general step sizes. We provide a complete description of the phase transitions. The phase transition turns out to depend on the summability of an appropriate sequence, rather than a critical value of the probability only. We exhibit novel scalings for certain critical cases. Time permitting, we shall also indicate a version of the process with binary steps where the memory is asymmetric. This is a joint work with Aritra Majumder.

Wednesday, 8 October 2025 (11:15-12:15)

Speaker : Moumanti Podder
Title : A model of market economics inspired by random walks with long memory

Imagine maintaining a chart for tracking the relative performance of one product that competes against another in an oligopolistic market. Every time an item of the first product is purchased, we add $a+1$ to our chart, and every time an item of the second product is purchased, we add $a-1$ to our chart, giving rise to a random-walk-like model where the relative performance of the first product with respect to the second, after a total of n items of these two products have been bought, is the same as the position of the walker that begins walking from the origin. In scenarios where customers may not have complete information regarding the underlying qualities of these two products, a customer may draw a sample of k customers from the past, and note the choices that were made by these sampled customers. Based on these sampled observations, the customer decides, according to a stochastic rule, which product to choose. This is reminiscent of a random walk where the walker decides every step based on their memory of the entire past, and can be thought of as a generalized version of the relatively new and extremely

popular random walk model known as the 'elephant random walk'. In this talk, I discuss results pertaining to strong and weak convergence of this generalized elephant random walk.

Wednesday, 8 October 2025 (14:15-15:15)

Speaker : Anirban Basak
Title : Large deviations in random graphs

Classical large deviation theory provides sharp asymptotics for probabilities of rare events involving linear statistics of independent random variables. A natural nonlinear extension arises in the study of Erdős–Rényi graphs, where one of the earliest and most tractable questions concerns large deviations of triangle counts. Over the past twenty years, this direction has led to substantial progress and new methods. In this talk, I will describe some of these developments, focusing in particular on large deviations for subgraph counts in sparse Erdős–Rényi graphs. Based on joint works with Riddhipratim Basu and Shaibal Karmakar.

Thursday, 9 October 2025 (09:30-10:30)

Speaker : Arijit Chakrabarty
Title : Top eigenvalues and eigenvectors of inhomogeneous Erdős-Rényi random graphs

The talk is on eigenvalues outside the spectrum of inhomogeneous ErdősRényi random graphs and the corresponding eigenvectors. Depending on the rank of the inhomogeneity kernel generating the random graph, the largest few eigenvalues have a much higher magnitude than that of the bulk. Assuming the rank to be finite, the second order behaviour of those few eigenvalues, after suitable centring and scaling, is shown to be multivariate Gaussian. The asymptotic behaviour of the corresponding eigenvectors is also studied.The talk is based on joint works with Bishakh Bhattacharya, Sukrit Chakraborty and Rajat Hazra.

Thursday, 9 October 2025 (14:15-15:15)

Speaker : Soumendu S. Mukherjee
Title : Elephants on Trees: A Non-Markovian Random Walk on Finitely Generated Non-Abelian Infinite Groups

We introduce a generalisation of Schütz and Trimper's Elephant Random Walk to finitely generated (non-abelian) infinite groups. Obtaining precise limit theorems for this model seems to be challenging due to the non-abelian and non-Markovian nature of the process. In this talk, we shall focus on the simplest setting of infinite Cayley trees and present some results on the escape rate and return probability.

Thursday, 9 October 2025 (16:00-17:00)

Speaker : Siva Athreya
Title : Interplay of vertex and edge dynamics for dense random graphs

The large population limits of opinion dynamics in homogeneous populations, on lattices and on general fixed graphs are quite well understood. We consider a process where the graph itself is dynamic and changes in response to the voter model process, thus creating interaction between the two. More precisely, we consider a dense random graph in which the vertices can hold opinion 0 or 1 and the edges can be closed or open. The vertices update their opinion at a rate proportional to the number of incident open edges, and do so by adopting the opinion of the vertex at the other end. The edges update their status at a constant rate, and do so by switching between closed and open with a probability that depends on their status and on whether the vertices at their ends are concordant or discordant. We understand the large n limit of this co-evolution and describe the limiting evolution. This is joint work with Frank den Hollander and Adrian Roellin.

Friday, 10 October 2025 (09:30-10:30)

Speaker : Tridib Sadhu
Title : How Singularities in Large Deviations Emerge in Stochastic Processes

Singularities in large deviation functions have been observed in a variety of stochastic systems, including experimental settings. In this talk, I will present a general mechanism that accounts for the emergence of such singularities across a broad class of processes. These singularities are shown to correspond to phase-transition-like phenomena in path space. To characterize these transitions, I will

discuss a framework based on generalized Doob transformations for Markov processes conditioned on empirical observables, leading to the notion of conditioned large deviations.

References:

- Large deviations conditioned on large deviations I: Markov chains and Langevin equations, Derrida and Sadhu, JSP, 176, 773 (2019).
- Large deviations conditioned on large deviations II: fluctuating hydrodynamics, Derrida and Sadhu, JSP, 177, 151 (2019).
- Dynamical phase transitions in certain non-ergodic stochastic processes, Reddy, Majumdar, Guiselin, and Sadhu, ArXiv: 2412.19516 (2024).

Friday, 10 October 2025 (11:15-12:15)

Speaker : Parthanil Roy
Title : Phase Transitions for Elephant Random Walks with Two memory Channels

Random processes with strong memory arise naturally in various disciplines including physics, economics, biology, geology, etc. Elephant random walk, introduced to study the effect of memory on random walks, is a special type of walk that incorporates the information of one randomly chosen past step to determine the future step. However, memory of a process can be multifaceted and can arise due to interactions of more than one underlying phenomena. To model this, random walks with multiple memory channels were introduced in the statistical physics literature by Saha (2022) - here the information of a bunch of independently chosen past steps is needed to decide the future step. The aforementioned work analyzed the two channel case and predicted phase transitions from diffusive to superdiffusive and from superdiffusive to ballistic regimes based on variance heuristics. We prove these conjectures rigorously (with mild corrections), discover a mildly superdiffusive regime at one of the conjectured transition boundaries, and observe new second-order phase transitions. We also carry out a detailed investigation of the asymptotic behavior of the walk at different regimes (based on a joint work with Krishanu Maulik and Tamojit Sadhukhan).