Discussion Meeting on
Ergodic Geometry

25 – 29 March, 2019

Schedule and Abstracts of Talks

School of Mathematics
Tata Institute of Fundamental Research

Title of Talks

Uri Bader	*Rigidity phenomena via Ergodic methods*
Sergio Fenley	*Partially hyperbolic diffeomorphisms*
Vladimir Finkelstein	*Measure rigidity for horospherical subgroups of groups acting on trees*
Alex Furman	*Rigidity phenomena via Ergodic methods*
Anders Karlsson	*Metric functionals and an ergodic theorem for noncommuting random products*
Seonhee Lim	*Local limit theorem in negative curvature*
Wenyuan Yang	*Floyd and Martin boundaries for a group*

Abstracts

Monday, 25 March 2019 (09:30-11:00)

Speaker : Uri Bader and Alex Furman
Title : Rigidity phenomena via Ergodic methods

Ideas from Dynamical Systems provide powerful tools for studying many problems in geometry and group theory. In this short series of lectures we will discuss a use of ergodic methods to prove some rigidity results for representations of higher-ran groups in hyperbolic like targets, and some applications of these ideas.

Monday, 25 March 2019 (11:30-13:00)

Speaker : Seonhee Lim
Title : Local limit theorem in negative curvature

The heat kernel $p(t,x,y)$ of the hyperbolic space H^n is explicitly known. The asymptotic formula of $p(t,x,y)$ as t goes to infinity was discovered for symmetric spaces by Bougerol. For random walks on Gromov hyperbolic groups, the analogous local limit theorem was proved by Gouzel. We prove local limit theorem of the heat kernel for the universal cover of a compact Riemannian manifold of negative curvature. In the first talk, we will give a short survey on local limit theorem of the heat kernel, then state our main theorems and the strategy of the proofs. In the second talk, we will explain where we use thermodynamics formalism for a Gibbs measure associated to the Green function. This is joint work with Franois Ledrappier.

Monday, 25 March 2019 (16:00-17:30)

Speaker : Wenyuan Yang
Title : Floyd and Martin boundaries for a group

In this lecture, I will discuss two topological boundaries of a finitely generated group obtained from rather different point of views. The Floyd boundary is constructed as the Cauchy completion of an exponential decay scaling of the Cayley graph. This serves in some sense a maximal boundary with a convergence action to cover geometrically finite actions of the group. In the first part of the lecture, I will explain some basic features of Floyd boundary and then construct a quasiconformal measure on Floyd boundary when the group is relatively hyperbolic. Applications are given to compute the Hausdorff dimension of limit sets of subgroups. This represents joint work with Leonid Potyagailo. In the second part of the lecture, Martin boundary will be constructed starting from a finitely supported measure on the group. This boundary can be integrated against Martin kernels with appropriate measures to represent all positive harmonic functions. When Martin kernels are understood as Busemann functions for Green's metric,

the Martin boundary is precisely the corresponding Busemann boundary. Relating to Floyd boundary, a relative Ancona inequality about Green's function is stated in terms of Floyd distance of three points. Using this inequality, I will explain how the identity map extends continuously as a surjection map from Martin to Floyd boundray. The surjection is injective on a generic class of points called conical points in Floyd boundray. This represents the joint work with Victor Gerasimov, Ilya Gehktman and Leonid Potyagailo.

Tuesday, 26 March 2019 (09:30-11:00)

Speaker : Uri Bader and Alex Furman
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Tuesday, 26 March 2019 (11:30-13:00)

Speaker : Anders Karlsson
Title : Metric functionals and an ergodic theorem for noncommuting random products

Metric functionals (a variant of Busemann functions) are to metric spaces what linear functionals are to linear spaces. In particular they define a weak topology with compactness properties. Invariant metrics appear in many places of mathematics, and it has turned out that often it is possible to have a description of metric functionals (compare with finding the dual spaces of Banach spaces). There is one general sort-of spectral statement that can be proven. This, when applied to various metric spaces, has in particular as application the Wolff-Denjoy theorem in complex analysis, the Carleman-von Neumann mean ergodic theorem and Thurstons spectral theorem for surface homeomorphisms. It is of interest even in the Banach space setting. Thanks to progress in sub additive ergodic theory, a corresponding ergodic theorem can also be established. The first version appeared in a joint work with Ledrappier, the more general one in joint work with Gouzel. Applicable to random walks on basically any group, in the particular setting of invertible matrices and symmetric spaces the result is equivalent to Oseledets multiplicative ergodic theorem.

Tuesday, 26 March 2019 (16:00-17:30)

Speaker : Sergio Fenley
Title : Partially hyperbolic diffeomorphisms

Hyperbolic or Anosov diffeomorphisms have contracting and expanding invariant bundles. These bundles are called stable and unstable bundles. These diffeomorphisms are extremely common and they have fantastic properties. However for a lot of questions one needs to consider a generalization called a partially hyperbolic diffeomorphism: in addition to the stable and unstable bundles, they have a center bundle which is invariant. The center vectors cannot expand more than the unstable vectors at the point and cannot contract more than the stable vectors at the point. One important example is the time one map of the geodesic flow on the unit tangent bundle of a hyperbolic surface. We will review some basic properties of Anosov diffeomorphisms, and then we will discuss partially hyperbolic diffeomorphisms. In particular we will concentrate more on dimension 3, where each of

the 3 bundles has dimension 1. There are questions of classification, integrability (the center bundle is not always integrable, and the center stable, center unstable bundles also), and rigidity; amongst others.

Wednesday, 27 March 2019 (09:30-11:00)

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Thursday, 28 March 2019 (09:30-11:00)

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Friday, 29 March 2019 (09:30-11:00)

Speaker : Vladimir Finkelstein

Title : Measure rigidity for horospherical subgroups of groups acting on trees

Let G be a locally compact group and $L < G$ a lattice. Conisder the action of $H < G$ on G/L. A fundamental question in homogeneous dynamics is to classify H-invariant measures on G/L. It was answered by Ratner when G is a linear group and H is a group generated by unipotent elements. We focus on non-linear groups acting on regular trees. If G is a subgroup of the group of automorphisms of a regular tree satisfying some technical conditions, $L < G$ a lattice, $H < G$ a horospherical subgroup, we can show that all H-invariant measures on G/L are homogeneous. Moreover, when L is uniform we obtain unique ergodicity of H-action. For lattices of Nagao type in $\mathrm{Aut}(T_d)$ we prove equidistribution and classify the orbit closures. This is joint work with Corina Ciobotaru and Cagri Sert.