Groups, Orbits and Diophantine Approximation

1 – 5 February, 2016

Schedule and Abstracts of Talks

The International Centre, Goa

Title of Talks

Jinpeng An	*Bounded orbits in the space of unimodular lattices*
Jayadev Athreya	*Diophantine approximation on translation surfaces*
Victor Beresnevich	*Sums of reciprocals of fractional parts and applications to Diophantine approximation (Part 2)*
Emmanuel Breuillard	*Metric diophantine approximation in matrices and on Lie groups*
Yann Bugeaud	*Exponents of Diophantine approximation*
Manfred Einsiedler	*Positive entropy and (multiple) quantitative unipotent recurrence*
Alexander Gorodnik	*Logarithmic improvements in Diophantine approximation*
Dmitry Kleinbock	*On Dirichlet's theorem for inhomogeneous approximation*
Philippe Michel	*On the second moment of twisted L-functions*
Frederic Paulin	*Counting and equidistribution of arithmetic points in local fields of positive characteristic*
Tali Pinsky	*On the volumes of modular geodesics*
Mark Policott	*Representations of surface groups, Higher Teichmuller theory and ergodic theory*
Nimish Shah	*On equidistribution of expanding translates of curves on homogeneous spaces*
Uri Shapira	*The distribution of lattices orthogonal to best approximations*
Andreas Strömbergsson	*On the low-density limit of the Lorentz gas for general scatterer configurations*
Nicolas de Saxce	*Diophantine approximation and product of linear forms*
Naser Talebi Zadeh	*Optimal strong approximation for quadratic forms*
Sanju Velani	*Sums of reciprocals of fractional parts and applications to Diophantine approximation (Part 1)*
Barak Weiss	*Badly approximable vectors on fractals*
Lei Yang	*Exponential mixing higher rank affine actions on S-adic nilmanifolds and smooth classification of higher rank expanding actions*

Abstracts

Monday, 1 February 2016 (09:00-10:00)

Speaker : Sanju Velani

Title : Sums of reciprocals of fractional parts and applications to Diophantine approximation (Part 1)

$$S_N(\alpha,\gamma) := \sum_{n=1}^{N} \frac{1}{n\|n\alpha-\gamma\|} \quad \text{and} \quad \mathrm{R}_{\mathrm{N}}(\alpha,\gamma) := \sum_{\mathrm{n}=1}^{\mathrm{N}} \frac{1}{\|\mathrm{n}\alpha-\gamma\|},$$

where α and γ are real parameters and $\|\cdot\|$ is the distance to the nearest integer. Our results improve upon previous results of W. M. Schmidt and others, and are (up to constants) best possible. Related to the above sums, we also obtain upper and lower bounds for the cardinality of

$$\{1 \le n \le N : \|n\alpha-\gamma\| < \varepsilon\},$$

valid for all sufficiently large N and all sufficiently small ε. This work is motivated by applications to multiplicative Diophantine approximation. In particular, we obtain complete Khintchine type results for multiplicative simultaneous Diophantine approximation on fibers in $\mathbb{R}^2$.

Monday, 1 February 2016 (10:30-11:30)

Speaker : Victor Beresnevich

Title : Sums of reciprocals of fractional parts and applications to Diophantine approximation (Part 2)

$$S_N(\alpha,\gamma) := \sum_{n=1}^{N} \frac{1}{n\|n\alpha-\gamma\|} \quad \text{and} \quad \mathrm{R}_{\mathrm{N}}(\alpha,\gamma) := \sum_{\mathrm{n}=1}^{\mathrm{N}} \frac{1}{\|\mathrm{n}\alpha-\gamma\|},$$

where α and γ are real parameters and $\|\cdot\|$ is the distance to the nearest integer. Our results improve upon previous results of W. M. Schmidt and others, and are (up to constants) best possible. Related to the above sums, we also obtain upper and lower bounds for the cardinality of

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valid for all sufficiently large N and all sufficiently small ε. This work is motivated by applications to multiplicative Diophantine approximation. In particular, we obtain complete Khintchine type results for multiplicative simultaneous Diophantine approximation on fibers in $\mathbb{R}^2$.

Monday, 1 February 2016 (11:30-12:30)

Speaker : Yann Bugeaud
Title : Exponents of Diophantine approximation

For an irrational real number x, let w(x) denote the supremum of the real numbers w such that there are arbitrarily large integers q with $\| qx \| < q^{-w}$, where $\| \cdot \|$ denotes the distance to the nearest integer. Let n be a positive integer. This definition can be extended to approximation to x by algebraic numbers of degree at most n, to small values of integer polynomials of degree at most n evaluated at x, or to simultaneous approximation to x, x^2, x^n by rational numbers with the same denominator. Moreover, we may as well introduce uniform exponents by replacing the requirement 'there are arbitrarily large integers' by 'for all sufficiently large integers' in the above definition. Thus, we have introduced six families of Diophantine exponents. We survey the relations between them and try to determine the sets of values taken by these exponents on the set of real numbers. We formulate several open questions and mention some recent advances.

Monday, 1 February 2016 (14:30-15:30)

Speaker : Jinpeng An
Title : Bounded orbits in the space of unimodular lattices

Given countably many one-parameter diagonalizable subgroups F_k of $SL(n,R)$, we expect that the set of points in the space of n-dimensional unimodular lattices such that all the F_k-orbits are bounded has full Hausdorff dimension. Work of Dmitry Kleinbock and Barak Weiss proves the validity of the expectation for $n = 2$. In a joint work with Lifan Guan and Dmitry Kleinbock, it is shown to be true for $n = 3$. In both cases, an essential tool is the notion of hyperplane absolute winning (HAW) set in a manifold. In arbitrary dimension, partial result is also obtained in work of Lifan Guan and Weisheng Wu.

Monday, 1 February 2016 (16:00-17:00)

Speaker : Mark Policott
Title : Representations of surface groups, Higher Teichmuller theory and ergodic theory

The study of Riemann surfaces can be formulated in terms of representations of the fundamental group in PSL(2,R). In higher Teichmuller theory one is interested in the representations of the fundamental group in PSL(d,R). One approach to generalizing familiar concepts for Riemann surfaces (e.g., Weil-Petersson metric, Selberg Zeta function) to these higher dimensional representations is to use ideas from ergodic theory. This is joint work with Richard Sharp (Warwick).

Tuesday, 2 February 2016 (09:00-10:00)

Speaker : Alexander Gorodnik
Title : Logarithmic improvements in Diophantine approximation

We discuss the problem of minimising products of (affine) linear forms computed at integral points and explain approach to this problem which is based on investigating quantitativerecurrence of orbits of higher-rank abelean groups. We will be interested in establishing quantitative (logarithmic) bounds on the products of forms. This is joint work with Pankaj Vishe.

Tuesday, 2 February 2016 (10:30-11:30)

Speaker : Emmanuel Breuillard
Title : Metric diophantine approximation in matrices and on Lie groups

A natural generalization of classical diophantine approximation consists in taking products of elements in a Lie group and ask how fast they can approach the identity. For example, classically, a number x is Diophantine if and only if sums of N terms each equal to 1, -1, x or -x, cannot get closer to zero than an inverse power of N. Replacing x and 1 by two or more vectors in a vector space is the subject of simultaneous approximation, and diophantine approximation in matrices. In this talk I will consider the case when the elements are chosen from a more general Lie group and focus on nilpotent groups. It turns out that the computation of optimal exponents for nilpotent Lie groups can be reduced to that of certain algebraic subvarieties of matrices. We give a formula for the exponent of arbitrary submanifolds of matrices, showing it depends only on their algebraic closure, answering a question of Kleinbock and Margulis, and use it to express the exponent of nilpotent Lie groups in terms of various representation theoretic data. This is joint work with M. Aka, L. Rosenzweig and N. de Saxce.

Tuesday, 2 February 2016 (11:30-12:30)

Speaker : Tali Pinsky
Title : On the volumes of modular geodesics

Consider a closed geodesic γ on the modular surface, embedded in the unit tangent bundle. The unit tangent bundle is the complement of a trefoil knot in S^3, and the geodesic is a knot therein. Its complement is always a hyperbolic three manifold, and thus has a well defined volume. I will show how to obtain an upper bound for this volume in terms of the length of γ. The main tools are the modular template and the continued fraction expansion corresponding to modular geodesics.

Wednesday, 3 February 2016 (09:00-10:00)
Speaker : Philippe Michel
Title : On the second moment of twisted L-functions

In this talk, we explain a solution to the long-standing and irritating problem of evaluating with a power saving error term the second moment of the central value of L-functions of modular forms twisted by Dirichlet characters of prime modulus. The proof is a combination of works of Blomer, Fouvry, Kowalski, Michel, Milicevic and Sawin combine three different type of arguments. Moreover the problem itself has a dynamical flavor which we will also discuss; in particular we will highlight an algebraic mixing conjecture whose solution would help to solve a number of important questions in analytic number theory.

Wednesday, 3 February 2016 (09:00-10:00)
Speaker : Manfred Einsiedler
Title : Positive entropy and (multiple) quantitative unipotent recurrence

We will discuss rigidity of positive entropy measures for higher rank diagonalisable actions. Using a quantitative form of recurrence along unipotent directions we prove a complete classification of positive entropy measures for any higher rank action on any irreducible quotients of SL_2^k as well as some other cases of interest. This is ongoing joint work with Elon Lindenstrauss.

Wednesday, 3 February 2016 (09:00-10:00)
Speaker : Jayadev Athreya
Title : Diophantine approximation on translation surfaces

We will survey how ideas and questions from diophantine approximation are used in the study of translation surfaces. Part of this will be joint work with Anish Ghosh and Dia Taha on generalizations of old problems of Erdos-Szusz-Turan and Kesten.

Wednesday, 3 February 2016 (09:00-10:00)

Speaker : Naser Talebi Zadeh

Title : Optimal strong approximation for quadratic forms

For a non-degenerate integral quadratic form $F(x_1,\dots,x_d)$ in 5 (or more) variables, we prove an optimal strong approximation theorem. Fix any compact subspace $\Omega\subset\mathbb{R}^d$ of the affine quadric $F(x_1,\dots,x_d)=1$. Suppose that we are given a small ball B of radius $0<r<1$ inside Ω, and an integer m. Further assume that N is a given integer which satisfies $N\gg(r^{-1}m)^{4+\varepsilon}$ for any $\varepsilon>0$. Finally assume that we are given an integral vector $(\lambda_1,\dots,\lambda_d)$ mod m. Then we show that there exists an integral solution $x=(x_1,\dots,x_d)$ of $F(x)=N$ such that $x_i\equiv\lambda_i$ mod m and $\frac{x}{\sqrt{N}}\in B$, provided that all the local conditions are satisfied. We also show that 4 is the best possible exponent. Moreover, for a non-degenerate integral quadratic form $F(x_1,\dots,x_4)$ in 4 variables we prove the same result if $N\geq(r^{-1}m)^{6+\varepsilon}$ and some non-singular local conditions for N are satisfied. Based on some numerical experiments on the diameter of LPS Ramanujan graphs, we conjecture that the optimal strong approximation theorem holds for any quadratic form $F(X)$ in 4 variables with the optimal exponent 4.

Wednesday, 3 February 2016 (09:00-10:00)

Speaker : Lei Yang

Title : Exponential mixing higher rank affine actions on S-adic nilmanifolds and smooth classification of higher rank expanding actions

We consider a Z^k action on an S-adic nilmanifold by affine automorphisms. We will prove an exponential mixing result on this action under a higher rank assumption. As an application, we prove a smooth classification result on higher rank expanding actions on compact manifolds. This is a joint work with Ralf Spatzier.

Thursday, 4 February 2016 (09:00-10:00)

Speaker : Frederic Paulin

Title : Counting and equidistribution of arithmetic points in local fields of positive characteristic

Given a function field K over a finite field and a completion K_p of K, we give several counting and equidistribution results of the points of K in K_p (analogous to Mertens' equidistribution result of Farey fractions), or the quadratic irrational elements of K_p over K in given orbits of $PGL(2,K)$. The techniques rely on the strong ergodic properties of the action of the discrete geodesic flow on the quotient of Bruhat-Tits trees by arithmetic lattices in $PGL(2,K_p)$. This is a joint work with A. Broise and J. Parkkonen.

Thursday, 4 February 2016 (10:30-11:30)

Speaker : Andreas Strömbergsson
Title : On the low-density limit of the Lorentz gas for general scatterer configurations

The Lorentz gas describes the dynamics of a cloud of non-interacting point particles in an infinite array of fixed spherical scatterers. We take the centers of the scatterers to be a fixed set P in R^d of constant asymptotic density. It was proved by Boldrighini, Bunimovich and Sinai that if P is a realization of a Poisson process with constant intensity then in the limit of low scatterer density (Boltzmann-Grad limit), the time evolution of a random initial particle is governed by the linear Boltzmann equation. Marklof and Strömbergsson proved a corresponding limit result when P is a lattice; in this case the limiting random flight process is more complicated. In the talk I will outline an approach to obtaining the limit for more general sets P, including for example special types of quasicrystals. A key assumption is the following: Let P_r be the random point set obtained by applying a random rotation to the fixed set P, followed by the linear transformation with matrix $\mathrm{diag}(r^(d-1), 1/r, ..., 1/r)$. As r tends to 0, this random point set P_r should converge in distribution to a point process satisfying certain properties. Joint work with Jens Marklof.

Thursday, 4 February 2016 (11:30-12:30)

Speaker : Barak Weiss
Title : Badly approximable vectors on fractals

In joint work with David Simmons, we show that the set of badly approximable vectors in R^d, are a measure zero set with respect to the natural self-similar measures on sufficiently regular fractals, such as the Koch snowflake or Sierpinski gasket. The proof uses a classification result for stationary measures on homogeneous spaces, extending work of Benoist and Quint

Thursday, 4 February 2016 (14:30-15:30)

Speaker : Dmitry Kleinbock
Title : On Dirichlet's theorem for inhomogeneous approximation

This will be a tale of two Diophantine problems: one easy to state and seemingly hard to solve, another (the inhomogeneous version of the first one) a bit more involved but with an elegant solution coming from dynamics on the space of lattices. The topic is Dirichlet's Theorem and its improvability, and our approach uses exponential mixing of a certain homogeneous flow. Joint work with Nick Wadleigh

Friday, 5 February 2016 (09:00-10:00)

Speaker : Nicolas de Saxce
Title : Diophantine approximation and product of linear forms

Fix a one-parameter diagonal flow a_t on the space of unimodular lattices in R^d, and consider its conjugates a_t^L, where L is chosen randomly on a subvariety M of $SL(d,R)$. Under some algebraicity condition on M, we will give a formula for the rate of escape of the orbit $a_t^L.Z^d$ and describe the shape of the lattice $a_t^L Z^d$, for t large and almost every L. Finally, we will give applications to classical problems of diophantine approximation on manifolds.

Friday, 5 February 2016 (10:30-11:30)

Speaker : Uri Shapira
Title : The distribution of lattices orthogonal to best approximations

I will discuss a work in progress with Barak Weiss in which we study the question of the existence and description of a limiting distribution of a sequence of lattices obtained by intersecting the integer lattice Z^d with the orthocomplements to a "naturally defined" sequence of integer vectors v_n. For example, given a random point p on the unit sphere in R^d, one can consider $\{v_n\}$ as the sequence of best approximations to the half-line $H_p = tp : t > 0$. It turns out that the orthogonal lattices have a limiting distribution but surprisingly enough, it is not the uniform one.

Friday, 5 February 2016 (11:30-12:30)

Speaker : Nimish Shah
Title : On equidistribution of expanding translates of curves on homogeneous spaces

We consider a finite piece C of an analytic curve on a minimal expanding (abelian) horospherical subgroup of $G = SL(n,R)$ associated to some element g in G. We consider the subgroup action of G on a finte volume homogeneous space X, and consider the trajectory of C from some point x in X. We want to find algebraic conditions on C which ensures that in the limit, the translates of Cx by powers of g get equidistributed in the closure of the G orbit from x. In this talk we describe some recent joint work with Lei Yang on this problem, and indicate its implication to some questions on metric properties of Diophantine approximation.