Title and Abstract

SINGULARITIES OF ADMISSABLE NORMAL FUNCTIONS

(REPORT ON JOINT WORK WITH MARK GREEN)

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Abstract

Normal functions provide a method for studying algebraic cycles inductively by their codimension. They were introduced by Poincaré and provided the technique for Lefschetz's proof of his $(1,1)$ theorem. Due to the failure of Jacobi inversion in higher codimension, they have been less useful in the study of cycles of higher codimension.

Normal functions are traditionally defined in a way that excludes singular behaviour. Recently it has been found that if one extends the concept of normal function to *admissible normal functions*, then there is an intricate relation between the singularities of these and algebraic cycles. Specifically, let (X^{2n}, L, ζ) be a smooth projective variety, L a very ample line bundle and $\zeta \in \mathrm{Hg}^n(X)_{\mathrm{prim}}$ a primitive Hodge class. Then there is an admissable normal function ν_ζ defined on the space S of hyperplane sections X_s of X.

Theorem. *The singular set* $\mathrm{sing}_{\nu_\zeta}$ *of* ν_ζ *is the subvariety given by*

$$\left\{ \begin{array}{c} s \in S : X_s \textit{ supports a new} \\ \textit{Hodge class in codimension } n-1 \end{array} \right\} .$$

A corollary is that the Hodge conjecture is equivalent to the statement

$$\mathrm{sing}_{\nu_\zeta} \text{ is non-empty for } L \gg 0 .$$

The proof of the final form of the above result is due to Brosnan-Fang-Pearlstein-Nie and independently to de Cataldo-Migliorini. It requires full use of the decomposition theorem of Beilinson-Bernstein-Deligne-Gabber applied to the special situation of the universal family of hyperplane sections of X fibered over S.

A second aspect regarding singularities of admissable normal functions concerns their realization as pullbacks of a universally defined

object as per the diagram

$$\begin{tikzcd} & \tilde{J}_{e,\Sigma} \supset \Xi \arrow[d] \\ S \arrow[ur, dashed, "\nu_\zeta"] \arrow[r, dashed, "\tau"] & \Gamma\backslash D_\Sigma \end{tikzcd}$$

where $\Gamma\backslash D_\Sigma$ are the Kato-Usui spaces, $\tilde{J}_{e,\Sigma}$ is a universal Néron model and Ξ is a subvariety. The realization of $\mathrm{sing}_{\nu_\zeta}$ would be expressed by

$$\mathrm{sing}_{\nu_\zeta} = \nu_\zeta^{-1}(\Xi) \ .$$

This represents work in progress and in particular requires (i) the construction of general Néron models for intermediate Jacobians over one dimensional base spaces, (ii) the extension of the classical Néron model to higher dimensional parameter spaces, and (iii) the combination of (i) and (ii). Here, (i) and (ii) have been done but (iii) has not.

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