Internationl Colloquium on
K-theory

6 – 14 January, 2016

Schedule and Abstracts of Talks

School of Mathematics
Tata Institute of Fundamental Research

Title of Talks

Aravind Asok	*Motivic vector bundles on projective spaces*
Samik Basu	*Twisted Homology theories*
Spencer Bloch	*Mixed motives and mixed Hodge structures associated to algebraic cycles*
Utsav Choudhury	*Motivic Galois groups and Standard conjectures*
Frédéric Déglise	*Dimensional homotopy t-structure*
Dan Edidin	*Strong regular embeddings of stacks and applications*
Hélène Esnault	*Chern classes of crystals*
Thomas Geisser	*Some remarks on etale motivic cohomology*
Christian Haesemeyer	*The K-theory of monoid algebras in mixed characteristic*
Masaki Hanamura	*Integrals of logarithmic forms on semi-algebraic sets and a generalized Cauchy formula*
Marc Hoyois	*Cdh descent for the homotopy K-theory of tame stacks*
Roy Joshua	*Equivariant Algebraic K-theory and Derived completion*
Bruno Kahn	*Cycles of codimension 2 and Chow-Knneth decompositions*
Wataru Kai	*A moving lemma for algebraic cycles with modulus and contravariance*
Amalendu Krishna	*Some K-cohomology of singular surfaces and applications*
Marc Levine	*Torsion indices of smooth projective varieties*
James Lewis	*The regulator map from Bloch's simplicial higher Chow groups to Deligne cohomology*
Arvind Nair	*Mixed motives in A_g*
Amnon Neeman	*Grothendieck duality via Hochschild homology*
Kapil Paranjape	*Modular Forms and Calabi-Yau varieties*
Jinhyun Park	*Algebraic cycles and crystalline cohomology*
Holger Reich	*Algebraic K-theory of group algebras and topological cyclic homology*
Oliver Röndigs	*The first motivic stable homotopy groups of spheres*
Andreas Rosenschon	*Torsion in the Lichtenbaum Chow group of arithmetic schemes*
Kay Rülling	*Higher Chow groups with modulus and relative Milnor K-theory*
Shuji Saito	*Motives with modulus*
Anand Sawant	*$\mathbb{A}^1$-connectedness in reductive algebraic groups*
Ramesh Sreekantan	*Cycles on Abelian surfaces*
Jean Louis Colliot-Thélène	*Stable irrationality for some rationally connected varieties : a survey*
Charles Weibel	*Relative Cartier Divisors and Laurent Polynomials*

Abstracts

Wednesday, 6 January 2016 (10:00-11:00)
Speaker : **Spencer Bloch**
Title : **Mixed motives and mixed Hodge structures associated to algebraic cycles**

This will be an expository talk focusing on the idea that algebraic cycles yield functions on the Tannaka group of mixed motives.

Wednesday, 6 January 2016 (11:30–12:30)
Speaker : **Aravind Asok**
Title : **Motivic vector bundles on projective spaces**

I will describe joint work with Mike Hopkins and Jean Fasel regarding construction of "motivic vector bundles" on P^n. By definition, a motivic vector bundle of rank r on P^n is an A^1-homotopy class of maps from P^n to BGL_r. Such classes correspond to algebraic vector bundles on any smooth affine variety that is isomorphic in the Morel-Voevodsky A^1-homotopy category to P^n (e.g., the variety X_n obtained as complement of the incidence hyperplane in the product of a projective space and its dual). In particular, I will explain how to construct indecomposable rank 2 bundles on X_n that "algebraize" a collection of rank 2 topological vector bundles on P^n described by Elmer Rees

Wednesday, 6 January 2016 (14:30–15:30)
Speaker : **Arvind Nair**
Title : **Mixed motives in A_g**

We will discuss the computation of some simple mixed motives appearing in the cohomology of the moduli space of principally polarized Abelian varieties and of its Satake compactification.

Wednesday, 6 January 2016 (16:00–17:00)
Speaker : **Utsav Choudhury**
Title : **Motivic Galois groups and Standard conjectures**

I will describe different constructions of motivic Galois groups due to Nori, Ayoub and Y. Andre. We will see that Noris motivic Galois group is canonically isomorphic to Ayoub's Galois group. Moreover, Andre's Galois group is the largest reductive quotient of the Galois group of Ayoub. In the end, we will see a reformulation of the standard conejctures using vanishing of Ayoub's Hopf d.g.a and conservative conjecture.

Speaker : **Marc Levine**
Title : **Torsion indices of smooth projective varieties**

This is a report on a joint work with Andre Chatzistimatiou. For a smooth projective variety X of dimension d over a field k, we consider the *torsion index* $\mathrm{Tor}_k X$ of X, this being the order (possibly infinite) of the diagonal in $\mathrm{CH}_d((X \setminus F_0) \times (X \setminus F^1))$, where F_0 is a sufficiently large dimension zero subset of X and F^1 is a sufficiently large codimension one subset of X (one takes F_0 and F^1 large enough so that the order of the diagonal remains constant under further enlargement). For X geometrically integral, we consider as well the *geometric torsion index* of X, $\mathrm{Tor}_{\bar{k}}(X_{\bar{k}})$ where $\bar{k}$ is the algebraic closure of k. The torsion index gives an upper bound for the exponent of unramified cohomology on X.

We consider the geometric torsion index of a complete intersection $X^{d_1,\ldots,d_r;n}$ of multi-degree $d_1 \geq \ldots \geq d_r \geq 2$ in $\P^{n+r}$ and the torsion index of a generic complete intersection in $\P^{n+r}$. The geometric construction of Roitman gives the upper bound

$$\mathrm{Tor}_{\bar{k}}(X_{\bar{k}}^{d_1,\ldots,d_r;n}) \leq \prod_{i=1}^{r} d_i!$$

if $\sum_i d_i \leq n+r$. Using the method of Kollár, Voisin, Colliot-Thélène-Pirutka and Totaro, we give a lower bound: Let $d_1 \geq \ldots \geq d_r \geq 2$ and $n \geq 3$ be integers such that $\sum_i d_i \leq n+r$. Let p be a prime number and suppose that

$$d_1 \geq p \cdot \left\lceil \frac{n+r+1-\sum_{i=2}^{r} d_i}{p+1} \right\rceil \tag{1}$$

Then $p|\mathrm{Tor}(X)$ for all very general $X = X_{d_1,\ldots,d_r} \subset \P^{n+r}$. Finally, we show that for the generic $X^{d_1,\ldots,d_r;n}$ in $\P^{n+r}$ (ie, the complete intersection defined by homogeneous equations with coefficients being independent variables $\ldots, \eta_i, \ldots$), we have the lower bound

$$\prod_{i=1}^{r} d_i!^* | \mathrm{Tor}_{k(\eta)} X^{d_1,\ldots,d_r;n}$$

Here, for a positive integer d, $d!^*$ is the least common multiple of the numbers $1,2,\ldots,d$. For example, the generic cubic hypersurface in $\P^{n+1}$ with $n \geq 2$ has torsion index equal to six, while the generic quartic hypersurface in $\P^{n+1}$ with $n \geq 3$ has torsion index either 12 or 24.

Thursday, 7 January 2016 (11:30–12:30)
Speaker   :   Jean Louis Colliot-Thélène
Title   :   **Stable irrationality for some rationally connected varieties
              : a survey**

In 1984, the first time I came to TIFR, I described examples of stably rational varieties which are not rational. A few years later, I lectured at TIFR on developments of the Artin-Mumford method, which enables one to prove stable irrationality. A new method, which involves Chow groups, was initiated by Claire Voisin in the end of 2013. It has witnessed quick developments, with application to stable irrationality of several types of hypersurfaces of low degree. I shall survey the method and its applications.

Thursday, 7 January 2016 (14:30–15:30)
Speaker   :   **Frédéric Déglise**
Title   :   **Dimensional homotopy t-structure**

At the center of Voevodsky's theory of motivic complexes over a field is the notion of homotopy invariant sheaves with transfers. They form the heart of a canonical t-structure on motivic complexes which reflects its relation with the coniveau filtration. This t-structure is fundamental for motivic complexes, and more broadly in motivic homotopy theory. It was later extended in several direction by Fabien Morel, Déglise and lastly Joseph Ayoub in the relative case. In collaboration with Bondarko, we propose a new definition of this t-structure over an arbitrary base relying on the notion of dimension function on schemes. A new important fact is that we also construct an effective version of this t-structure, therefore staying very close to Voevodsky's original motivic complexes. During the talk, I will give the definition and state the main properties which have many common features with the perverse t-structure. Our main theorem is a characterization of the t-structure in terms of "fibers" over points of the base. Consequently, the heart of this t-structure is a convenient extension of the category of h.i. sheaves with transfers. In the end, I will give some of its main properties as well as some computations related with singularities, abelian schemes and relative curves.

Thursday, 7 January 2016 (16:00–17:00)
Speaker   :   **Ramesh Sreekantan**
Title   :   **Cycles on Abelian surfaces**

In this talk we use generalizations of classical geometric constructions of Kummer and Humbert to construct new higher Chow cycles on Abelian surfaces and K3 surfaces over p-adic local fields, generalising some work of Collino. The existence of these cycles is predicted by the poles of the local L-factor at p of the L-function of the Abelian surface. The techniques involve using some recent work of Bogomolov, Hassett and Tschinkel on the deformations of rational curves on K3 surfaces. As an application we use these cycles to prove an analogue of the Hodge-D-conjecture for Abelian surfaces

Friday, 8 January 2016 (10:00–11:00)

Speaker : **Shuji Saito**
Title : **Motives with modulus**

I will report on a joint work with Bruno Kahn and Takao Yamazki on the construction of a triangulated category of motives with modulus that extends Voevodsky's category in such a way as to encompass non-homotopy invariant phenomena

Friday, 8 January 2016 (11:30–12:30)

Speaker : **Marc Hoyois**
Title : **Cdh descent for the homotopy K-theory of tame stacks**

The cdh topology on schemes is the topology generated by Nisnevich covers and abstract blowup squares. C. Haesemeyer (in characteristic zero) and D.-C. Cisinski (in general) have shown that Weibel's homotopy invariant K-theory satisfies cdh descent. I will discuss an extension of Cisinski's result to quotient stacks of schemes by linear actions of linearly reductive algebraic groups

Friday, 8 January 2016 (14:30–15:30)

Speaker : **Oliver Röndigs**
Title : **The first motivic stable homotopy groups of spheres**

In joint work with Markus Spitzweck and Paul Arne Østvaer, we study the spectral sequence based on Voevodsky's slice filtration. This filtration on the stable homotopy category of motivic spectra over a field F measures the amount of Tate suspensions which are necessary to construct a given motivic spectrum. Work of Levine and Voevodsky shows that the slices of the motivic sphere spectrum are determined by the second page of the topological Adams-Novikov spectral sequence. We use this information to compute the first stable motivic homotopy groups of spheres over fields of characteristic zero, at least up to a completion with respect to the first algebraic Hopf map

Friday, 8 January 2016 (16:00–17:00)

Speaker : **Wataru Kai**
Title : **A moving lemma for algebraic cycles with modulus and contravariance**

The theory of algebraic cycles with modulus is an emerging branch of algebraic cycle theory. Examples are the additive higher Chow group introduced by Bloch, Esnault and Park, and the higher Chow group with modulus by Binda, Kerz and Saito. The groups are believed to correspond to relative K-groups of pairs of a smooth variety and a Cartier divisor on it. In this talk we exhibit how the contravariance of these theories can be deduced from a "moving lemma with modulus".

Saturday, 9 January 2016 (10:00–11:00)
Speaker   :   Charles Weibel
Title   :   Relative Cartier Divisors and Laurent Polynomials

This is joint work with Vivek Sadhu. If A is a subring of B (or more generally $f : X \to Y$ is affine faithful) there is a natural determinant from the relative K0 group to the group of relative Cartier divisors, reducing to the classical case when A is normal with quotient field B. We study the behavior of relative divisors under polynomial and Laurent polynomial extensions. This is a contracted functor in Bass' sense, and the contraction is the group of global sections of the etale sheaf $f_*(Z)/Z$.

Saturday, 9 January 2016 (11:30–12:30)
Speaker   :   Bruno Kahn
Title   :   Cycles of codimension 2 and Chow-Knneth decompositions

I will review known results and outstanding questions on algebraic cycles, concentrating on the case of codimension 2, and explain how refined Chow-Kňneth decompositions shed a light on these questions, yield a few new results and raise some new questions.

Saturday, 9 January 2016 (14:30–15:30)
Speaker   :   Kapil Paranjape
Title   :   Modular Forms and Calabi-Yau varieties

The classical result of Eichler and Shimura associates, to each eigenform of weight 2 with rational coefficients, a corresponding elliptic curve. We will discuss how this may be generalisable to other weights by using Calabi-Yau varieties of higher dimension. We will also present some examples. This talk is based on joint work with Dinakar Ramakrishnan

Saturday, 9 January 2016 (16:00–17:00)

Speaker : **Masaki Hanamura**
Title : **Integrals of logarithmic forms on semi-algebraic sets and a generalized Cauchy formula**

We consider integrals of the form

$$\int_A \frac{dz_1}{z_1} \wedge \cdots \wedge \frac{dz_n}{z_n}$$

where A is a compact semi-algebraic set of dimension n in *mathbbC^n*. (1) Under a certain condition on the dimension intersection of A with the "faces" (intersections of coordinate hyperplanes), the integral is shown to be convergent. (2) The "Cauchy formula" for integrals as above are formulated is formulated and proven. (3) One can apply (1) and (2) to the study of mixed Tate Hodge structures associated to mixed Tate motives. These are meant to further part of the work of Bloch and Kriz on mixed Tate motives. This is joint work with K. Kimura and T. Terasoma.

Monday, 11 January 2016 (10:00–11:00)

Speaker : **Thomas Geisser**
Title : **Some remarks on etale motivic cohomology**

We will give some results (well-known to the experts) on motivic cohomology over algebraically closed fields, and discuss properties of duality pairings between mod m and the m-torsion of etale motivic cohomology over algebraically closed, finite and local fields.

Monday, 11 January 2016 (11:30–12:30)

Speaker : **Kay Rulling**
Title : **Higher Chow groups with modulus and relative Milnor K-theory**

For a pair (X,D) consisting of a smooth variety X over a field and an effective Catier divisor D, S. Saito and K. Kato defined in the 80's a relative version of Milnor K-theory in order to study higher dimensional geometric class field theory. Recently M. Kerz and S. Saito gave a different approach using Chow groups of zero cycles of (X,D). In this talk I will explain a link between these two approaches in the case where the support of D is a simple normal crossings divisor. Namely, in this case we construct a cycle map from the motivic complex of (X,D) in weight r, introduced by S. Saito and F. Binda, to the relative Milnor K-sheaf of (X,D) in degree r and shifted by [-r]. This map induces an isomorphism between the Nisnevich cohomology groups in degrees greater equal r+ dim X. This is joint work with S. Saito

Monday, 11 January 2016 (14:30–15:30)
Speaker : Roy Joshua
Title : Equivariant Algebraic K-theory and Derived completion

Derived completion is a technique that originated slightly over 20 years ago. However, Gunnar Carlsson may have been the first to realize the true potential of this technique, especially in the context of Algebraic K-theory. The goal of this talk is to discuss some applications of this technique, especially in the context of our joint work with Carlsson, to equivariant algebraic K-theory

Monday, 11 January 2016 (16:00–17:00)
Speaker : Samik Basu
Title : Twisted Homology theories

The study of twisted homology and cohomology generalises the definition of twisted K-theory by Karoubi and later studied by Atiyah, Segal and Freed, Hopkins and Teleman, The definition may be exteded to arbitrary commutative ring spectra. Apart from geometric considerations, the theory is amenable to detection of ring structures in module spectra and computation of invariants like topological Hochschild homology.

Tuesday, 12 January 2016 (10:00–11:00)
Speaker : Hélène Esnault
Title : Chern classes of crystals

The crystalline Chern classes of the value on the characteristic p variety of a torsion-free crystal depend only the isocrystal class, and vanish if the crystal is locally free or the isocrystal is convergent. It is conjectured that they always vanish. One deduces from it new cases of de Jong conjecture on the relation between the ètale fundamental group and the category of isocrystals. This is joint work with Atsushi Shiho, Tokyo University

Tuesday, 12 January 2016 (11:30–12:30)
Speaker : Christian Haesemeyer
Title : The K-theory of monoid algebras in mixed characteristic

Given a commutative monoid M, the natural numbers act on M via dilations, that is, via power maps. Gubeladze conjectured that if M is contained in a lattice and contains no non-trivial units, and if R is any regular commutative ring, then the dilations act nilpotently on the reduced K-theory of the monoid algebra $R[M]$. Using trace methods, this had previously been proved when R contains a field. In this talk on joint work with G. Cortiñas, M. Walker and C. Weibel, we explain how to prove the conjecture in the general case, building on our proof in positive characteristic

Tuesday, 12 January 2016 (14:30–15:30)
Speaker : **Jinhyun Park**
Title : **Algebraic cycles and crystalline cohomology**

I will talk about a description of the sheaf of big de Rham-Witt complexes on smooth varieties in terms of certain algebraic cycle groups, called additive higher Chow groups. Its degree 0 part gives an intersection-theoretic description of the ring structure of the ring of big Witt vectors, which is notorious for being complicated. In addition we deduce a description of the crystalline cohomology in terms of algebraic cycles. (joint work with Amalendu Krishna)

Tuesday, 12 January 2016 (16:00–17:00)
Speaker : **James Lewis**
Title : **The regulator map from Bloch's simplicial higher Chow groups to Deligne cohomology**

An explicit formula for the Bloch cycle class map from his higher Chow groups to Deligne cohomology, was provided by Kerr/Lewis/Müller-Stach [KLM] (Compositio Math 142) for projective algebraic manifolds, and in the general case by Kerr-Lewis (invent. Math. (2007)), based on a cubical description of these groups. We provide an explicit formula for the simplicial version of these groups. This is based on joint work with Matt Kerr and Patrick Lopatto, with an appendix by Jose Burgos Gil.

Wednesday, 13 January 2016 (10:00–11:00)
Speaker : **Amnon Neeman**
Title : **Grothendieck duality via Hochschild homology**

Hochschild cohomology was introduced in a 1945 paper by Hochschild and Grothendieck duality dates back to the early 1960s. The fact that the two have some relation with each other is very new - it came up in papers by Avramov and Iyengar [2008], Avramov, Iyengar, and Lipman [2010] and Avramov, Iyengar, Lipman and Nayak [2011]. We will review this history, and the surprising formulas that come out. We will then discuss more recent progress. The remarkable feature of all this is the role played by Hochschild homology. One example, which we will discuss in some detail, comes about as follows. The new techniques permit us to write formulas giving trace and residue maps in Grothendieck duality in terms of expressions that are very Hochschild-homological - Alonso, Jeremias and Lipman gave such a formula, but couldn't prove that it agrees with the usual formula dating back to Verdier in the 1960s. The proof that these two agree, due to Lipman and the speaker, turns out to hinge on considering the action of ordinary Hochschild homology on the various objects in the formula.

Wednesday, 13 January 2016 (11:30–12:30)

Speaker : **Anand Sawant**

Title : $\mathbb{A}^1$**-connectedness in reductive algebraic groups**

Invariants of algebraic groups such as the Whitehead group and R-equivalence have played an important role in the study of near-rationality properties of algebraic groups. Classical results about algebraic groups combined with recent works in motivic homotopy theory allow one to relate these invariants with $\mathbb{A}^1$-connectedness in certain simply connected algebraic groups. However, the picture is very different for algebraic groups that are not simply connected. We will discuss some recent results in this direction. The talk is based on joint work with Chetan Balwe.

Wednesday, 13 January 2016 (14:30–15:30)

Speaker : **Holger Reich**

Title : **Algebraic K-theory of group algebras and topological cyclic homology**

The Whiteheadgroup $Wh(G)$ and its higher analogues defined using algebraic K-theory play an important role in geometric topology. There are vanishing conjectures in the case G is torsionfree. For groups containing torsion the Farrell-Jones conjecture gives a conjectural description in terms of group homology. I will report on a new result which for example allows to detect a large direct summand inside the rationalized Whiteheadgroup of a group like Thompson's group T. The proof uses the cyclotomic trace to topological cyclic homology, Boekstedt-Hsiang-Madsen's functor C, and new general injectivity results about the assembly maps for THH and C. There are interesting relations to the Leopoldt-Schneider conjecture. The talk will report on joint work with Wolfgang Lueck (Bonn), John Rognes (Oslo) and Marco Varisco (Albany)

Wednesday, 13 January 2016 (16:00–17:00)

Speaker : **Andreas Rosenschon**

Title : **Torsion in the Lichtenbaum Chow group of arithmetic schemes**

We give an example of a smooth scheme over the spectrum of a Dedekind domain such that the torsion subgroup of a Lichtenbaum Chow group is infinite. This is joint work with V. Srinivas.

Thursday, 14 January 2016 (10:00–11:00)
Speaker : Dan Edidin
Title   : Strong regular embeddings of stacks and applications

We explain how the notion of strong regular embedding can be used to compare the K-theory of a Deligne-Mumford stack to that of a regularly embedded substack. This can be applied to deduce results about K-theory of schemes in a number of interesting examples, such as the relationship between the K-theory of a hypertoric variety and its corresponding Lawrence toric variety. Although this talk is about stacks, the motivating examples come from observations about invariant rings for actions of finite groups.

Thursday, 14 January 2016 (11:30–12:30)
Speaker : Amalendu Krishna
Title   : Some K-cohomology of singular surfaces and applications

We study the divisibility property of the cohomology groups of certain K-theory sheaves on singular surfaces and give some applications to algebraic cycles.