# Annual Discussion Meeting on
# Complex Analytic Geometry

**26 – 30 March, 2018**

# Schedule and Abstracts of Talks

**School of Mathematics**

**Tata Institute of Fundamental Research**

## Title of Talks

**Gautam Bharali**      *An interpolation problem viewed through the lens of reductive group action on Stein domains*

**Usha Bhosle**      *Embedding of a compactified Jacobian*

**Arijit Dey**      *Equivariant principal bundles on toric varieties*

**Sorin Dumitrescu**      *Quasihomogeneous analytic geometric structures*

**Subhojoy Gupta**      *Harmonic maps and wild Teichmüller spaces*

**Sebastian Heller**      *Deformations of Deligne-Hitchin moduli spaces and sections thereof*

**Amit Hogadi**      *Gabber presentation lemma for finite fields*

**Jacques Hurtubise**      *Monads for instatnons and bows*

**Niky Kamran**      *A survey of non-uniqueness results for the anisotropic Calderón problem with disjoint data*

**Hisashi Kasuya**      *Techniques of constructions of variations of mixed Hodge structures*

**Marina Logares**      *A lax TQFT on representation varieties*

**Takuro Mochizuki**      *Periodic monopoles and difference modules*

**Vamsi Pingali**      *An extension of a theorem of Nori to Gauduchon astheno-Kahler manifolds*

**Florent Schaffhauser**      *Hitchin components for fundamental groups of 2-orbifolds*

**Georg Schumacher**      *Moduli of Logarithmic Pairs - Analytic Theory*

# Abstracts

### *Monday, 26 March 2018 (10:00-11:00)*
**Speaker   :   Takuro Mochizuki**
**Title      :   Periodic monopoles and difference modules**

One of the main themes in complex geometry is to obtain a correspondence between differential geometric objects and algebro-geometric objects. In this talk, we shall explain a kind of Kobayashi-Hitchin correspondence between singular periodic monopoles of GCK type and stable parabolic difference modules

### *Monday, 26 March 2018 (11:30-12:30)*
**Speaker   :   Vamsi Pingali**
**Title      :   An extension of a theorem of Nori to Gauduchon astheno-Kahler manifolds**

Nori proved that a vector bundle on a projective variety satisfies a polynomial equation with integral coefficients if and only if its pullback to a finite cover is trivial. I shall talk about an extension of this result to Gauduchon astheno-Kahler manifolds. This is joint work with Indranil Biswas. Since the Kobayashi-Lubke inequality plays an important role in our proof, if time permits I shall make some comments on how one might hope (based on evidence from the line bundle case due to Collins-Xie-Yau) to find equations (like the Hermite-Einstein equation) whose solutions, if they exist, might lead to an inequality involving higher Chern classes.

### *Monday, 26 March 2018 (14:30-15:30)*
**Speaker   :   Sebastian Heller**
**Title      :   Deformations of Deligne-Hitchin moduli spaces and sections thereof**

The Deligne-Hitchin moduli space $\mathcal{M}_{DH}$ is a certain compactification of the moduli space of $\lambda$-connections on a Riemann surface. It can be interpreted as the twistor space of the hyper-Khler moduli space of solutions of Hitchins selfduality equations by the work of C. Simpson. In my talk I will discuss deformations of $\mathcal{M}_{DH}$ induced by deforming the conformal type or the genus of the underlying surface. Such deformations give rise to deformations of sections of the Deligne-Hitchin moduli space, at least in general. We explain how this method can be used to construct counter examples of a question of C. Simpson. This talk is based on joint work with L.Heller.

### Tuesday, 27 March 2018 (10:00-11:00)

**Speaker** : **Niky Kamran**

**Title** : **A survey of non-uniqueness results for the anisotropic Calderón problem with disjoint data**

After giving a general introduction to the main known results on the anisotropic Calderón problem on $n$-dimensional compact Riemannian manifolds with boundary, we shall give a motivated review of some recent non-uniqueness results for the anisotropic Calderón problem at fixed frequency, in dimension $n \geq 3$, when the Dirichlet and Neumann data are measured on disjoint subsets of the boundary. These non-uniqueness results are of the following nature: given a smooth compact connected Riemannian manifold with boundary $(M, g)$ of dimension $n \geq 3$, we first show that there exist in the conformal class of $g$ an infinite number of Riemannian metrics $\tilde{g}$ such that their corresponding Dirichlet-to-Neumann maps at a fixed frequency coincide when the Dirichlet data $\Gamma_D$ and Neumann data $\Gamma_N$ are measured on disjoint sets and satisfy $\overline{\Gamma_D \cup \Gamma_N} \neq \partial M$. The corresponding conformal factors satisfy a nonlinear elliptic PDE of Yamabe type on $(M, g)$ and arise from a natural but subtle gauge invariance of the Calderón when the data are given on disjoint sets. We then present counterexamples to uniqueness in dimension $n \geq 3$ to the anisotropic Calderón problem at fixed frequency with data on disjoint sets, which do not arise from this gauge invariance. They are given by cylindrical Riemannian manifolds with boundary having two ends, equipped with a suitably chosen warped product metric. Time permitting, we will make some remarks on the case of manifolds with corners. This is joint work with Thierry Daude (Cergy-Pontoise) and Francois Nicoleau (Nantes).

### Tuesday, 27 March 2018 (11:30-12:30)

**Speaker** : **Florent Schaffhauser**

**Title** : **Hitchin components for fundamental groups of 2-orbifolds**

Let $Y$ be a compact connected 2-orbifold of negative Euler characteristic and let $\pi$ be its orbifold fundamental group. For $n > 1$, we denote by $R(\pi, n)$ the space of representations of $\pi$ into $PGL(n, R)$. The purpose of the talk is to show that $R(\pi, n)$ possesses a connected component homeomorphic to an open ball whose dimension we can compute explicitly (for $n = 2$ and 3, we find again formulae due to Thurston and to Choi and Goldman, respectively). We then give several applications of the result. This is joint work with Daniele Alessandrini and Gye-Seon Lee (University of Heidelberg).

### Tuesday, 27 March 2018 (14:30-15:30)

**Speaker** : **Amit Hogadi**

**Title** : **Gabber presentation lemma for finite fields**

In this talk I will outline the proof of Gabber's presentation lemma over finite fields. This is joint work with Girish Kulkarni.

*Wednesday, 28 March 2018 (10:00-11:00)*
**Speaker** : **Jacques Hurtubise**
**Title** : **Monads for instatnons and bows**


Instantons on the Taub-NUT space are related to 'bow solutions' via a generalization of the ADHM-Nahm transform. Both are related to complex geometry, either via the twistor transform or via the Kobayashi-Hitchin correspondence. We explore various aspects of this complex geometry, exhibiting equivalences. For both the instanton and the bow solution we produce two monads encoding each of them respectively. Identifying these monads we establish the one-to-one correspondence between the instanton and the bow solution.(Joint work with Sergey Cherkis).


*Wednesday, 28 March 2018 (11:30-12:30)*
**Speaker** : **Subhojoy Gupta**
**Title** : **Harmonic maps and wild Teichmüller spaces**


The Teichmller space $T$ of a closed surface $S$ of genus greater than or equal to two is the space of marked hyperbolic structures on $S$. In the 1980s M.Wolf and N. Hitchin independently showed that $T$ can be parametrized by holomorphic quadratic differentials, with respect to a choice of complex structure, on $S$. I shall describe how this parametrization can be extended to a correspondence between meromorphic quadratic differentials with higher order poles, and the Teichmller space of crowned hyperbolic surfaces. The proof involves showing the existence of a harmonic map from a punctured Riemann surface to a crowned hyperbolic surface, with prescribed principal parts of its Hopf differential, which determine the asymptotic behaviour of the map near the punctures.


*Wednesday, 28 March 2018 (14:30-15:30)*
**Speaker** : **Hisashi Kasuya**
**Title** : **Techniques of constructions of variations of mixed Hodge structures**


The purpose of this talk is to give a way of constructing real variations of mixed Hodge structures (R-VMHS) over compact Kahler manifolds by using mixed Hodge structures on Sullivans 1-minimal models. This construction is very similar to known ideas (e.g Hain-Zucker). But this may be essentially different from them, since the obtained R-VMHS depends on Kahler metric. It is expected that this construction gives a differential geometric view of R-VMHSs.

**Speaker** : **Usha Bhosle**
**Title** : **Embedding of a compactified Jacobian**

Using a general stable vector bundle, we give an embedding of the compactified Jacobian $\bar{J}(Y)$ of an integral nodal curve $Y$ into the moduli space $U_Y(r,d)$ of semistable torsion free sheaves of rank $r$ and degree $d$ on $Y$. We also give an embedding of the normalisation $\widetilde{J(Y)}$ of $\bar{J}(Y)$ in the normalisation $P(r,d)$ of $U_Y(r,d)$. We determine a relation between the restriction of the theta line bundle on $P(r,d)$ to $\widetilde{J(Y)}$ and the theta line bundle on $\widetilde{J(Y)}$. We show that the restriction of the Picard bundle $E_{r,d}$ on $U_Y(r,d)$ to $\bar{J}(Y)$ is stable with respect to any theta divisor $\theta_{\bar{J}(Y)}$ on $\bar{J}(Y)$ if $d > r(2g-1)$ and it is semistable if $d = r(2g-1)$

*Monday, 26 March 2018 (11:30-12:30)*
**Speaker** : **Marina Logares**
**Title** : **A lax TQFT on representation varieties**

In this talk I will show the construction of a lax TQFT based on the geometric computational methods for the e-polynomials of a character variety. In order to do so I will introduce the so called Hodge monodromy representation in terms of Saitos mixed Hodge modules. This is joint work with Angel González-Prieto, and Vicente Muňoz.

*Thursday, 29 March 2018 (14:30-15:30)*
**Speaker** : **Gautam Bharali**
**Title** : **An interpolation problem viewed through the lens of reductive group action on Stein domains**

$H^\infty$ control theory is a branch of the theory of robust control of systems comprising interconnected devices each of whose outputs depend linearly on the inputs. Classical diagnostics for deciding whether such a system can be stabilised, given uncertainty in its parameters, are now recognised to be overly conservative. One approach to developing better diagnostics rigorously involves some interesting geometric invariant theory (GIT) in the Stein category. Both the classical and the contemporary quests for diagnostics reduce to certain Pick–Nevanlinna-type interpolation problems: into the "unit ball" – relative to a homogeneous functional called the structured singular value – in the modern view. Such a "unit ball" is an unbounded, (usually) non-hyperbolic Stein domain. From the work of Agler–Young in the early-2000s (who make no mention of GIT) one is led to suspect that the modern interpolation problem – for interpolation data in general position – is equivalent to an interpolation problem on a hyperbolic domain of much lower dimension. It turns out that the latter domain is a GIT quotient of the above-mentioned "unit ball" under the action of a reductive complex Lie group. That such a quotient exists follows from the work of Snow and collaborators on Stein

GIT quotients. In this talk, we shall first elaborate upon this and describe a family of these GIT quotients. Not every set of generators of the invariant ring – whose prime spectrum is the GIT quotient of interest – is useful to the question of interpolation that we have in mind. We shall describe a special set of generators using which, together with analytical methods, we shall establish the conjectured equivalence between the relevant interpolation problems.

**Speaker  :  Georg Schumacher**
**Title        :  Moduli of Logarithmic Pairs - Analytic Theory**

Let $X$ be a compact complex manifold and $D$ a smooth (or snc) divisor. We assume trhat the logarithmic pair $(X, D)$ is canonically polarized i.e. $K_X + D > 0$. Moduli spaces exists, and the complements $X\ D$ possess complete Kaehler-Einstein metrics. We show that the variation of these metrics give rise to a Kaehler structure on the moduli space. The Kaehler forms are shown to be the curvature forms of certain (positive, hermitian) determinant line bundles $L$. We show that these can be extended as positive currents to suitable compactifications of the moduli spaces, hence the hermitian line bundles can be extended to a compactification. In particular these are ample modulo the compactifying divisor. The latter fact is also related to a recent result with M. Paun.

**Speaker  :  Arijit Dey**
**Title        :  Equivariant principal bundles on toric varieties**

Following Nori, torus (T) equivariant principal G bundles over a toric variety X are precisely the functors from Gmod to the category of Tequivariant vector bundles satisfying certain conditions. Assuming the base field to be complex numbers, we will prove a Klyachko type theorem for the classification of Tequivariant principal Gbundles over X with at most factorial singularities, when G is reductive. This is based on joint work with Indranil Biswas and Mainak Poddar.

**Speaker  :  Sorin Dumitrescu**
**Title        :  Quasihomogeneous analytic geometric structures**

This talk deals with analytic geometric structures which are quasihomogeneous, in the sense that they are locally homogeneous on an open dense subset of a manifold, but not on all of the manifold. Our motivation comes from Gromov's open-dense orbit theorem and its application to prove the differential rigidity of some smooth Anosov systems and from a result of Bogomolov dealing with holomorphic tensors on complex manifolds with algebraic dimension zero. We will present the classification of quasihomogeneous real analytic connections on surfaces (collaboration with A. Guillot) and the case of real analytic Lorentz metrics on threefolds (collaboration with K. Melnick). We will also present the corresponding classification results on complex manifolds.