Discussion Meeting on
" Bundles - 2024 "

25 March - 29 March, 2024

Abstracts of Talks

School of Mathematics

Tata Institute of Fundamental Research

Title of Talks

Usha Bhosle	*Picard bundles and twisted Picard bundles on curves.*
Anoop Singh	*On the moduli space of Lie algebroid connections over a curve*
Ritwik Mukherjee	*Tangency as a limit of transverse intersections*
Utsav Choudhury	$\mathbb{A}^1$-*Homotopy type of moduli stacks*
Krishna Hanumanthu	*Seshadri constants on blow-ups of Hirzebruch surfaces*
Debojyoti Bhattacharya	*On l-away ACM bundles*
Arijit Dey	*Brauer group of moduli of parabolic G- bundles*

Abstracts

Tuesday, 26 March 2024 (11:30-12:30)

Speaker : **Usha Bhosle**
Title : **Picard bundles and twisted Picard bundles on curves.**

Let Y denote an irreducible projective curve with at most nodes as singularities and defined over an algebraically closed field of characteristic zero. We study Picard bundles and Twisted Picard bundles on the compactified Jacobian $\bar{J}(Y)$ of Y.

We define an embedding α_Y of the (generalised) Jacobian $J(Y)$ in the moduli space $U_Y^s(n,d)$ of stable vector bundles on Y using a twisted restriction E_Y of a Picard bundle to Y embedded in $J(Y)$ (by Abel-Jacobi map). We show that (under suitable conditions) the restriction of the universal bundle $\mathcal{U}$ on $J(Y) \times J(Y)$ to $Y \times J(Y)$ is stable for a suitable polarisation. For Y a smooth curve, we show that the restriction of the Picard bundle on $U_Y^s(n,d)$ to $J(Y)$ is θ-semistable (or θ-stable) for certain values of d. We also determine the relation between the restriction of the theta divisor on $U_Y^s(n,d)$ to $J(Y)$ and the theta divisor θ on $J(Y)$.

We study the restriction of the twisted Picard bundles on the compactified Jacobian $\bar{J}(Y)$ of Y to the embedded curve in $\bar{J}(Y)$. As an application, we show that for $g = 2$ and each integer $r \geq 3$, there is a two-dimensional family of stable ACM bundles on the compactified Jacobian which has the Picard bundle in its limit.

Tuesday, 26 March 2024 (14:30-15:30)

Speaker : **Anoop Singh**
Title : **On the moduli space of Lie algebroid connections over a curve**

We discuss some algebro-geometric properties of the moduli space of holomorphic Lie algebroid connections on a compact Riemann surface. We establish a smooth compactification for the moduli space of Lie algebroid connections such that the underlying vector bundle is stable. The complement of the moduli space into its compactification yields a divisor. We give a criterion for the numerical effectiveness of the divisor. We compute the Picard group of the moduli space, and analyse Lie algebroid Atiyah bundles associated with an ample line bundle. This enables us to conclude that the regular functions on the moduli space of certain Lie algebroid connections are constants. We also explore the rational connectedness of this moduli space.

Wednesday, 27 March 2024 (11:30-12:30)

Speaker : **Ritwik Mukherjee**
Title : **Tangency as a limit of transverse intersections**

Enumerative geometry deals with the following question: "How many geometric objects are there that satisfy certain constraints?" An important class of enumerative question is as follows: how many curves are there that satisfy certain constraints and are tangent to a given divisor? This is a very important question that has been studied by several mathematicians and is still an exciting problem due to its connections with Relative Gromov-Witten invariants. In this talk, we will give a very intuitive way to interpret tangency: we will view tangency as a limit of two transverse intersections. Using this idea, we will show how to solve enumerative problems that involve tangencies. If time permits, we will also indicate how our idea can be applied in the setting of stable maps and how it can enable us to compute Relative Gromov-Witten invariants. This is joint work with Indranil Biswas, Apratim Choudhury and Anantadulal Paul.

Wednesday, 27 March 2024 (14:30-15:30)

Speaker : **Utsav Choudhury**
Title : $\mathbb{A}^1$**-Homotopy type of moduli stacks**

In this lecture following [1], [2], [3], [4] we will survey the constructions of $\mathbb{A}^1$-homotopy types of the moduli stacks. Then we will see few applications of these constructions. If time premits we will see few possible questions that one can further probe.

Reference

[1] U. Choudhury, N. Deshmukh, A. Hogadi, *The Nisnevich motive of an algebraic stack, Annals of K-Theory*, Volume 8, Year 2023, Pages 245–273.

[2] U. Choudhury, *Motives of Deligne-Mumford stacks. Advances in Mathematics*, 231, 2012, 6, 3094–3117.

[3] A. Hogadi, S. Yadav, $\mathbb{A}^1$- *connectivity of moduli of vector bundles on a curve*, Journal of Institute of Math of Jussieu.

[4] V. Hoskins, S. Pepin Lehalleur, *On the Voevodsky motive of the moduli stack of vector bundles on a curve*, Quart. J. Math. (2021) 71 (1-2), 71–114

Speaker   :   **Krishna Hanumanthu**
Title   :   **Seshadri constants on blow-ups of Hirzebruch surfaces**

Seshadri constants measure local positivity of line bundles on projective varieties. They have connections to many interesting problems and are an active area of current research. We will first discuss a few important questions and results in this area and then talk about some recent results giving bounds on Seshadri constants for line bundles on blow-ups of Hirzebruch surfaces. This talk is based on joint work with Cyril Jacob, Suhas B. N., and Amt Kumar Singh

Thursday, 28 March 2024 (11:30-12:30)
Speaker   :   **Debojyoti Bhattacharya**
Title   :   **On l-away ACM bundles**

ACM bundles on a smooth, projective variety can be considered as vector bundles having the simplest possible cohomology (to be more precise, they are vector bundles without intermediate cohomology). The origin of these bundles dates back to the famous splitting theorems by Grothendieck and Horrocks for vector bundles over $\mathbb{P}^1$ (and $\mathbb{P}^n$ respectively). In this talk, we will first recall a brief history of ACM and Ulrich (a particular kind of ACM bundles) bundles, followed by their importance and some interesting questions related to them. As the next natural step in the non-ACM path, we will then move on to discuss the 'l-away ACM' bundles which were recently introduced by Gawron and Genc. After briefly mentioning the existing literature, we will discuss certain results on l-away ACM line bundles over cubic surfaces and l-away ACM bundles higher rank on $\mathbb{P}^2$. This is based on the ongoing joint works in progress with A.J. Parameswaran and Jagadish Pine.

Thursday, 28 March 2024 (14:30-15:30)
Speaker   :   **Arijit Dey**
Title   :   **Brauer group of moduli of parabolic G- bundles**

Let k be an algebraically closed field of characteristic zero. We prove that the Brauer group of the moduli stack of stable parabolic $\mathrm{PGL}(r,k)$-bundles with full flag quasi-parabolic structures at an arbitrary parabolic divisor on a curve X coincides with the Brauer group of the smooth locus of the corresponding coarse moduli space of parabolic $\mathrm{PGL}(r,k)$-bundles. We also compute the Brauer group of the smooth locus of this coarse moduli for more general quasi-parabolic types and weights satisfying certain conditions. This is a joint work with Indranil Biswas and Sujoy Chakraborty.