

**Conference on
Cohomology of Arithmetic Groups**

December 28-31, 2011

Schedule and Abstracts of Talks

**Tata Institute of Fundamental Research
Homi Bhabha Road, Mumbai 400 005**

DAY/DATE	9.30 a.m.-10.30 a.m.	T	11.00 a.m.-12.00 p.m.	12.00 p.m. - 1.00 p.m.	L	2.30 p.m.-3.30 p.m.	T	4.00 p.m.-5.00 p.m.
Wednesday, Dec. 28	L. Saper ^{@@}		Mahan Mj ^{@@}	Arvind Nair [@]	U	N. Bergeron ^{@@}		J. Tilouine ⁺⁺
Thursday, Dec. 29	A. Lubotzky ^{@@}	E	Kumar Murty ^{@@}	Vaibhav Vaish ^{@@}	N	J. Rohlf ^{@@}	E	A. Yafaev ^{@@}
Friday, Dec. 30 [—]	L. Clozel ⁺⁺		M. Harris ⁺⁺	B. Klingler ⁺⁺	C	TBA		Felicitatation Programme⁺⁺
Saturday, Dec. 31	T. Kobayashi ^{@@}	A	B. Speh ^{@@}	M. Rapoport ^{@@}	H		A	

The lectures will be held in Lecture Theatre (AG-66)^{@@} & Lecture Room (AG-69)⁺⁺,

Title of Talks

- L. Saper** *Raghunathan's Vanishing Theorem and Applications.*
- Mahan Mj** *Discreteness of Commensurators.*
- Arvind Nair** *Mixed Hodge(-de Rham) structures and arithmetic groups.*
- N. Bergeron** *Hodge type theorems for the cohomology of arithmetic groups of orthogonal type.*
- J. Tilouine** *Overconvergent Igusa tower and overconvergent p -adic Siegel modular forms.*
- A. Lubotzky** *Arithmetic groups, Ramanujan graphs and error correcting codes.*
- Kumar Murty** *TBA.*
- Vaibhav Vaish** *Motivic realization of intersection complexes on compactified Siegel modular varieties.*
- J. Rohlfs** *Lefschetz numbers and Bianchi groups.*
- A. Yafaev** *A hyperbolic Ax-Lindemann theorem in the cocompact case.*
- L. Clozel** *A proof of the Burger-Li-Sarnak conjecture for the Laplacian eigenvalues on congruence hyperbolic varieties.*
- M. Harris** *Gross-Prasad periods and values of L -functions.*
- B. Klingler** *Symmetric differentials and fundamental group.*
- T. Kobayashi** *Discrete spectrum for non-Riemannian Locally Symmetric Spaces.*
- B. Speh** *Restrictions of representations of semisimple Lie groups and the cohomology of arithmetic groups, an overview.*
- M. Rapoport** *On formal moduli spaces of p -divisible groups.*

Title of Talks

Wednesday, 28 December 2011 (9.30-10.30)

Speaker : L. Saper
Title : **Ragunathan's Vanishing Theorem and Applications.**

TBA

Wednesday, 28 December 2011 (11.00-12.00)

Speaker : Mahan Mj
Title : **Discreteness of Commensurators.**

In the first part of the talk, we shall show that commensurators of Zariski dense subgroups of isometry groups of symmetric spaces of non-compact type are discrete provided that the limit set on the Furstenberg boundary is not invariant under the action of a (virtual) simple factor.

In the second part, we prove that for all finitely generated, Zariski dense, infinite covolume discrete subgroups of $\text{Isom}(\mathbb{H}^3)$, commensurators are discrete.

Time permitting we shall show that it follows that arithmetic hyperbolic 3-manifolds fibering over the circle have infinite virtual first betti number.

Wednesday, 28 December 2011 (12.00-1.00)

Speaker : Arvind Nair
Title : **Mixed Hodge(-de Rham) structures and arithmetic groups**

The cohomology of a Hermitian locally symmetric space carries a natural mixed Hodge (or Hodge-de Rham) structure, which is pure if the variety is compact. We will discuss several results about these mixed structures and their relation to automorphic forms. Some of these results are new and some are old (in which case the proofs are new), but they are all proved using the same method of weight truncations (of S. Morel).

Wednesday, 28 December 2011 (2.30-3.30)

Speaker : N. Bergeron
Title : **Hodge type theorems for the cohomology of arithmetic groups of orthogonal type.**

In 1981 Millson and Ragunathan showed that the standard arithmetic examples of arithmetic groups $\Gamma \subset SO(p, q)$ ($p \geq q$) may have nonzero cohomology in degree q . In particular: the vanishing theorem of Borel-Wallach is the best possible.

Such nonzero cohomology classes were constructed using totally geodesic submanifolds. In the last months John Millson, Colette Moeglin and I were able to show that if $p + q > 6$ these totally geodesic submanifolds SPAN the $(p - 1)q$ -th homology of the manifold M associated to Γ . This is proved by combining work of Steve Kudla and John Millson from the 1980's where they constructed closed forms dual to these totally geodesic submanifolds using the Weil representation with work of Jim Arthur on the classification of automorphic representations of classical groups and the Siegel-Weil theorem as extended by Kudla-Rallis, Moeglin and Ginzburg-Jiang-Soudry in more recent years. These last two pieces of work are used to prove ALL the q -th cohomology comes from this Weil representation construction. There are corresponding results for homology (with coefficients) of degree $(p - n)q$, $0 < n < (p + q - 3)/4$.

I will try to avoid technical details and explain the overall principles. I will also describe some geometric and number theoretic consequences of our work.

Wednesday, 28 December 2011 (4.00-5.00)

Speaker : **J. Tilouine**

Title : **Overconvergent Igusa tower and overconvergent p -adic Siegel modular forms.**

Let $g \geq 1$ and p be a prime; let X/\mathbb{Q}_p be a Siegel variety of genus g and level prime to p ; let X^{rig} be the associated rigid space. Let $X^{\text{rig}}(\nu)$ (for $\nu > 0$) be a strict neighborhood of the ordinary locus $X^{\text{rig}}(0)$ in X^{rig} . Brinon and Mokrane defined a profinite étale covering $T_{\infty, \nu} \rightarrow X^{\text{rig}}(\nu)$ generalizing the Igusa tower over the ordinary locus.

In a joint work with them, we give a new definition of overconvergent p -adic Siegel modular forms as certain functions on this tower, and we compare our forms to other notions of Siegel modular forms (classical or p -adic). In this way, we reconstruct the eigenvariety for $GSp(2g)$.

Thursday, 29 December 2011 (9.30-10.30)

Speaker : **A. Lubotzky**

Title : **Arithmetic groups, Ramanujan graphs and error correcting codes.**

While many of the classical codes are cyclic, a long standing conjecture asserts that there are no 'good' cyclic codes. In recent years, interest in symmetric codes has been stimulated by Kaufman, Sudan, Wigderson and others (where symmetric means that the acting group can be any group). Answering their main question (and contrary to common expectation), we show that there DO exist symmetric good codes. In fact, our codes satisfy all the "golden standards" of coding theory.

Our construction is based on the Ramanujan graphs constructed by Lubotzky-Samuels-Vishne as a special case of Ramanujan complexes. The crucial point is that these graphs are edge transitive and not just vertex transitive as in previous constructions of Ramanujan graphs. These complexes are obtained as quotients of the Bruhat-Tits building modulo the action of suitable arithmetic groups. We will discuss the potential of these complexes and their cohomology to yield more applications to coding theory. All notions will be explained. Joint work with Tali Kaufman. Joint work with Tali Kaufman.

Thursday, 29 December 2011 (11.00-12.00)

Speaker : Kumar Murty
Title : TBA

TBA

Thursday, 29 December 2011 (12.00-1.00)

Speaker : Vaibhav Vaish
Title : **Motivic realization of intersection complexes on compactified Siegel modular varieties.**

We discuss the problem of realizing the intersection complexes extending homogeneous local systems on compactifications of Siegel modular varieties of dimension 3 in a motivic manner, i.e. using smooth projective varieties and algebraic cycles. The aim is to show that the cohomology of the discrete L^2 spectrum of the arithmetic group $Sp(4, \mathbb{Z})$ with coefficients in certain representations is (Chow) motivic, generalizing a result of Scholl and Deligne for $SL(2, \mathbb{Z})$.

Thursday, 29 December 2011 (2.30-3.30)

Speaker : J. Rohlfes
Title : **Lefschetz numbers and Bianchi groups.**

Let σ be an automorphism of finite order on the locally symmetric space $\Gamma \backslash X$ associated to an arithmetic group Γ . We assume that σ also acts on a finite dimensional representation V of Γ such that $\sigma(\gamma v) = \sigma(\gamma)\sigma(v)$ for all $v \in V$ and $\gamma \in \Gamma$. Then the Lefschetz number $L(\sigma, \Gamma, V) = \sum_i (-1)^i \text{tr}(\sigma | H^i(\Gamma, V))$ is defined.

If Γ is torsion free there are known methods to compute $L(\sigma, \Gamma, V)$ explicitly. If Γ has torsion there is a filtration of the fixed point set of σ in $\Gamma \backslash X$ depending on the set of σ -stable Γ -conjugacy classes of isotropy groups of Γ . In terms of this filtration the computation of $L(\sigma, \Gamma, V)$ is reduced to the computation of Euler-Poincaré characteristics of certain σ -stable locally symmetric subspaces. Let

$\Gamma = SL_2(O_d)/\{\pm 1\}$ be a Bianchi modular group, i.e. $k = \mathbb{Q}(\sqrt{-d})$, $3 < d \in \mathbb{N}$, with ring O_d of integers of k . An explicit formula for $L(\sigma, \Gamma, V)$ is obtained. Here V is the self-dual irreducible representation of $SL_2(\mathbb{C})$ with $\dim_{\mathbb{C}} V = (n+1)^2$. The formula for $L(\sigma, \Gamma, V)$ depends in an elementary way on the prime divisors of d and it is essentially linear in n . It implies estimates of the dimension of the cuspidal cohomology with coefficients V .

Thursday, 29 December 2011 (4.00-5.00)

Speaker : A. Yafaev
Title : **A hyperbolic Ax-Lindemann theorem in the cocompact case.**

This work is motivated by J.Pila's strategy to prove the Andre-Oort conjecture. One ingredient in the strategy is the following conjecture: Let S be a Shimura variety uniformised by a symmetric space X . Let V be an algebraic subvariety of S . Maximal algebraic subvarieties of the preimage of V in X are precisely the components of the preimages of weakly special subvarieties contained in V . We will explain the proof of this conjecture in the case where S is compact.

Friday, 30 December 2011 (9.30-10.30)

Speaker : L. Clozel
Title : **A proof of the Burger-Li-Sarnak conjecture for the Laplacian eigenvalues on congruence hyperbolic varieties.**

This is common work with N. Bergeron.

Friday, 30 December 2011 (11.00-12.00)

Speaker : M. Harris
Title : **Gross-Prasad periods and values of L -functions.**

Let G be a reductive group over \mathbb{Q} associated to a Shimura variety. The motive $M(\pi)$ defined by an automorphic representation π of G of cohomological type admit several realizations. The de Rham realization in the coherent cohomology of automorphic vector bundles is the closest to the classical theory of modular forms. Canonical models of automorphic vector bundles give rise to rational structures on these coherent cohomology spaces, and using the Beilinson-Bernstein theory of localization these structures extend to a (de Rham) rational structure on the entire representation π , over an appropriate number field E . It is often very useful to be able to characterize the E -rational elements in π by analytic means. When G is a unitary similitude group, the Gross-Prasad conjecture, slightly strengthened at real places, defines analytic period invariants that can be used for such a characterization, providing a vast generalization of Shimura's theory of CM periods for arithmetic holomorphic automorphic forms. The Ichino-Ikeda conjecture for unitary groups relates these Gross-Prasad periods to special values of L -functions.

Friday, 30 Friday 2011 (12.00-1.00)

Speaker : B. Klingler
Title : **Symmetric differentials and fundamental group**

Classical Hodge theory says that the sheaves of exterior differentials control the usual Betti cohomology of a smooth complex projective variety X . Is there a similar relation between the sheaves of symmetric differentials and the topology of X

? In this talk I will explain how symmetric differentials control finite dimensional representations of the topological fundamental group of X . Using automorphic forms this leads to new striking rigidity properties for fundamental groups of certain ball quotients.

Saturday, 31 December 2011 (9.30-10.30)

Speaker : T. Kobayashi

Title : Discrete spectrum for non-Riemannian Locally Symmetric Spaces.

I will discuss

1. discontinuous groups Γ for symmetric spaces X with indefinite metric.
2. spectral analysis of the Laplacian on X/Γ . In this situation, the Laplacian is no more an elliptic operator. Taking an odd dimensional ant-de Sitter manifold and a three dimensional “indefinite projective space” as examples, I plan to illustrate a new idea for the construction of L^2 spectrum which is stable under the deformation of discontinuous groups.

Saturday, 31 December 2011 (11.00-12.00)

Speaker : B. Speh

Title : Restrictions of representations of semisimple Lie groups and the cohomology of arithmetic groups, an overview.

TBA

Saturday, 31 Decmeber 2011 (12.00-1.00)

Speaker : M. Rapoport

Title : On formal moduli spaces of p -divisible groups.

After recalling the existence theorem of Zink and myself of the moduli spaces in question, I will give examples and state a recent comparison result of Kudla and myself.