Revised First Year Syllabus

(effective from Academic year 2020-21)

Guidelines

- The duration of each course is two semesters consisting of around 30 weeks of lectures in total.
- The syllabus is divided into prerequisites, core syllabus and some additional/optional topics. It is expected that the students are familiar with the prerequisites before joining. Some of the prerequisites may be briefly reviewed during the courses.
- The core syllabus is expected to constitute roughly 75% of the lectures. It is expected that the entire core syllabus is covered.
- The remaining part of the course will be as per the discretion of the instructor. Some possible additional topics are listed (after the core syllabus in the Algebra and Topology syllabi, with an asterisk * in the Analysis syllabus).
- Some reference books are listed for the core syllabus as well as additional topics. However, the choice of references used in a particular course solely lies with the instructor. The instructors are free to choose reference material not listed in the syllabus.

Algebra

Prerequisites:

- Groups: Definitions and examples, homomorphisms, subgroups, normal subgroups, Lagrange's theorem, isomorphism theorems, group actions, semi-direct products, Sylow's theorems, fundamental theorem of finitely generated abelian groups.
- Linear Algebra: Systems of linear equations, basic facts about vector spaces and linear transformations, eigenvalues and eigenvectors, matrices, determinants, ranks of linear transformations, characteristic and minimal polynomials, symmetric matrices, inner products, positive definiteness, Jordan and rational canonical forms, diagonalizable linear transformations.
- Rings and Modules: Commutative rings, ideals, prime and maximal ideals, integral domains and fields, Chinese remainder theorem, prime and irreducible elements, Unique Factorization Domains, Principal Ideal Domains, Euclidean Domains, Gauss's Lemma, Eisenstein's Criterion, polynomial rings, modules over rings, quotients, isomorphism theorems, modules over PID.
- Fields: Definitions and examples; characteristic of a field; algebraic extensions; normal and separable extensions.

Core syllabus:

- Groups: Simple groups; symmetric and alternating groups; free groups; nilpotent and solvable groups; Jordan-Holder theorem.

- Rings and Modules: Noetherian and Artinian rings and modules; semisimple rings; Artin-Wedderburn theorem; tensor products of modules; tensor, symmetric and exterior algebras; free modules, flat, projective and injective modules.

- Representation theory of finite group: Complete reducibility; characters and orthogonality; Group algebras; Induction and Mackey theory.

- Field and Galois theory: Algebraic extensions; separable and inseparable extensions; Galois theory and applications; infinite Galois theory; transcendental basis.

- Homological Algebra: Categories and functors; adjoint functors; additive and abelian categories; Hom and Tensor and their exactness properties and derived functors.

- Commutative Algebra: Integral extensions; Hilbert basis theorem; Noether normalisation theorem; Hilbert's Nullstellensatz; localization; discrete valuation rings and Dedekind domains and some applications to arithmetic; primary decomposition; dimension theory; completions.

References for the core syllabus:

- Algebra by S. Lang
- A First Course in Noncommutative Rings by T. Y. Lam
- TIFR Notes on Semisimple Rings and Central Simple Algebras
- Linear Representations of Finite Groups by J. P. Serre, Parts I & II
- Representation Theory by Fulton & Harris, Part I
- Introduction to Commutative Algebra by Atiyah & MacDonald
- An Introduction to Homological Algebra by C.A. Weibel

Some possible additional topics:

- Brauer groups and central simple algebras
- Group cohomology
- Quadratic forms
- Introduction to modular representations
- Introduction to Lie algebras and their representations
- Regular local rings
- Spectral sequences
- Introduction to derived and triangulated categories
- Any other topic at the discretion of the instructor

References for the additional topics:

- Central simple algebras and Galois cohomology by Gille & Szamuely
- Linear Representations of Finite Groups by J. P. Serre, Part III
- Representation Theory by Fulton & Harris, Part II
- Commutative Algebra by H. Matsumura
- An Introduction to Homological Algebra by C.A. Weibel
- Methods of Homological Algebra by Gelfand & Manin

Topology

Prerequisites:

Basic general and metric topology: connectedness, compactness, proper maps, quotient space construction and quotient topology, normal and Hausdorff spaces, paracompact spaces.

Core Syllabus:

- Examples: Spheres, quotients by properly discontinuous action, real and complex projective spaces, Grassmannians, Lens spaces, CW complexes, basic facts about topology of CW complexes, CW structures for standard examples, topological groups and continuous actions.

- Fundamental groups: Covering spaces; homotopy of maps, homotopy equivalence of spaces, contractible spaces, deformation retractions; fundamental group: universal cover and lifting problem for covering maps; Van Kampen's theorem, Galois coverings.

- Homology theory: Simplicial complexes, barycentric subdivision, simple approximations, singular homology, basic properties - excision, Mayer-Vietoris, cellular homology, singular cohomology, cup and cap product, Poincaré duality, Lefschetz fixed point theorem, Kunneth formula, universal coefficient theorem.

- Smooth manifolds: Tangent and cotangent spaces, differential forms, the de Rham theorem, vector fields, integral curves, the Frobenius theorem.

References for the core syllabus:

- Spanier: Algebraic topology
- Hatcher: Algebraic topology
- Massey: Algebraic topology: an introduction
- May: A concise course in algebraic topology
- Vick: Homology theory
- Bott-Tu: Differential forms
- Milnor: Topology from the differentiable viewpoint
- Warner: Foundations of Differentiable Manifolds and Lie Groups

Some possible additional topics:

- Topics in higher homotopy theory
- Spectral sequences in algebraic topology
- Topics in Riemannian geometry
- Topics in differential topology
- Morse theory
- Sheaf cohomology
- Characteristic classes
- Obstruction theory

- Categorical homotopy theory
- Spectra and basic stable homotopy theory
- Any other topic at the discretion of the instructor

References for the additional topics:

- Whitehead: Elements of homotopy theory
- Guillemin-Pollack: Differential topology
- Milnor: Morse theory
- Do Carmo: Riemannian geometry
- Milnor-Stasheff: Characteristic classes
- Switzer: Algebraic topology - homology and homotopy
- Griffiths-Morgan: Rational homotopy theory
- Hilton: General Cohomology Theory and K-Theory
- Hirschhorn: Model categories and their localizations
- Adams: Stable homotopy and generalized cohomology

Analysis

Prerequisites:

- Functions: Riemann integration, properties of continuous functions, sequences and series of functions and their different types of convergence, differentiable functions on $\mathbb{R}^n$, Implicit Function Theorem, Inverse Function Theorem, Taylor's Theorem.
- Topological notions: Definition and basic properties of topological spaces including but not limited to metric spaces, Bolzano-Weierstrass theorem, Heine-Borel theorem, Hausdorffness, connectedness and path connectedness, compactness, completion of metric spaces, convergence in metric spaces, continuity of functions between topological spaces.
- Linear algebra: Basic facts about vector spaces, matrices, determinants, ranks of linear transformations, diagonalizable linear transformations, inner products.

Syllabus: Topics marked with an asterisk * are optional.

- Elementary Functional Analysis: Topological vector spaces, Banach spaces, Bounded linear transformations, linear functionals, dual spaces, Hilbert spaces, Hahn-Banach theorem, Open mapping theorem, Uniform Boundedness Principle.

- Measure Theory: Abstract theory, convergence theorems, Product measure, the Fubini theorem, Hölder's inequality, Minkowski inequality. L^p-spaces, their completeness, their duals, Complex measures, Differentiation and decomposition of measures, the Radon-Nikodym Theorem, Borel measures on a locally compact space, regularity properties of measure, the Riesz Representation Theorem, the Lebesgue measure, *Differentiation: Maximal functions, *the Lebesgue differentiation theorem, *functions of bounded variation, Topological groups and Haar measure. Examples and basic properties. Unimodularity, *Measures on factor groups and homogeneous spaces.

- Elementary Harmonic Analysis: Convolutions, approximate identity (on $\mathbb{R}$ or on locally compact topological groups), *Approximation theorems, Fourier Transform, Fourier inversion formula and Plancherel theorem on $\mathbb{R}$ or $\mathbb{R}^n$, *Hausdorff-Young inequality.

- Operator Theory: Hilbert-Schmidt operators, Spectral theorem for compact self-adjoint and normal operators. *Banach algebras, *the Gelfand-Naimark theorem for commutative C*-algebras, *Continuous functional calculus, Spectral theorem for bounded self-adjoint/normal operators, *Peter-Weyl theorem for compact topological groups, *Representations of compact Lie groups, *Fourier transform for locally compact abelian groups, *Pontryagin duality, *Gelfand-Naimark-Segal construction.

- Complex Analysis: Cauchy-Riemann equations, holomorphic functions and their basic properties, Open mapping theorem, maximum modulus theorem, Zeroes of holomorphic functions, Schwarz lemma, Compactness and convergence in the space of analytic functions, normal families, Schwarz reflection principle, Weierstrass factorization theorem, *Zeroes of real analytic functions, *Riemann mapping theorem, Meromorphic functions, essential singularities, Picard's theorem, Analytic continuation, monodromy theorem, Hyperbolic metric on the disk/upper half-plane, and isometries/conformal automorphisms, *Compact Riemann Surfaces and the Riemann-Roch theorem.

- * Distribution Theory: The topological vector spaces of test functions and Schwartz functions associated to an open subset of $\mathbb{R}^n$, their duals. Convolution and Fourier transform for distributions. Paley-Wiener theorems. Fundamental Solutions of constant coefficient partial differential operators. Sobolev spaces, Elliptic regularity.

- * Ergodic Theory: Mean ergodic theorem.

- * Several Complex Variables: Basic theory of several complex variables. Domains of convergence of power series, domains of holomorphy. Weierstrass preparation theorem. Hartog's extension phenomenon. Biholomorphic inequivalence of the ball and the polydisk. Definition of complex manifolds, calculus on complex manifolds. Holomorphic vector bundles, divisors, and line bundles.

- Any other optional topic at the discretion of the instructor.

References

- Real and Complex Analysis, Walter Rudin.
- Functional Analysis, Walter Rudin
- Essential results in functional analysis, Robert J. Zimmer.
- Real and Functional Analysis by Serge Lang
- Principles of Harmonic Analysis, Anton Deitmar and Siegfried Echterhoff
- An invitation to C*-algebras, William Arveson.
- Fourier Analysis on Groups, Walter Rudin.
- Complex Analysis In the Spirit of Lipman Bers, Jane P Gilman, Irwin Kra and Rubí E Rodríguez.
- Complex Analysis, Elias M Stein and Rami Shakarchi.
- Complex Analysis, Theodore W Gamelin.
- Complex Analysis, Lars V Ahlfors.
- Complex Analysis in One Variable, Raghavan Narasimhan and Yves Nievergelt.
- Algebraic Curves and Riemann Surfaces, Miranda.
- Foundations of Ergodic Theory, Krerley Oliveira and Marcelo Viana.
- An Introduction to Ergodic Theory, Peter Walters.
- An Introduction to Complex Analysis in Several Variables, Lars Hörmander
- Holomorphic Functions and Integral Representations in Several Complex Variables, R. Michael Range.
- Complex Geometry, Daniel Huybrechts.