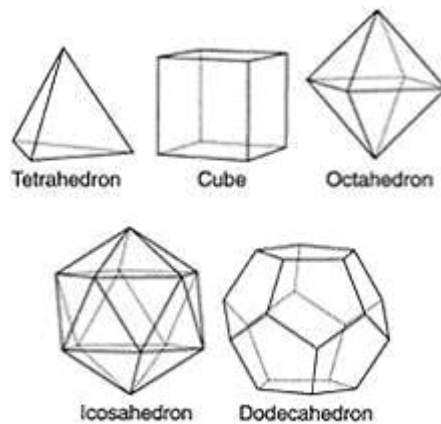


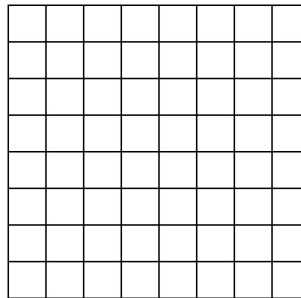
Groups actions and applications

Problem set - 1

1. Find the number of rotational symmetries as well as all symmetries (including reflections) of all the platonic solids:



2. In how many different ways can you colour all the 64 squares in the following 8×8 board using 2 colours, say black and white. (Two colourings of the board are considered to be the same if you can obtain one from the other by rotating the board.)



3. Let P be a group of order p^a , where p is prime. Prove that for every $b \in \{0, 1, \dots, a\}$, P has a normal subgroup of order p^b .
4. Let G be a finite group and let p be the smallest prime number dividing $|G|$. Suppose there exists a subgroup $H \leq G$ of index p . Then prove that H must be normal in G .
Hint: One way is to study the action of H on G/H by left multiplication. What can the orbits of this action look like?