Online short courses in VV-2021, Mathematics
Title and Abstracts

1. Tanmay Deshpande: Group actions and some applications

Abstract: We will study the notion of a group acting on a set and see some examples and applications. We will see how group actions can be used to prove Sylow's Theorems. We will also study some important classes of groups like nilpotent, solvable and simple groups.

2. T. N. Venkataramana: Topics in Number Theory

Abstract: Using the zeta function and analogue (Dirichlet L functions), we give a proof of infinitude of primes in arithmeticprogressions (Dirichlet's theorem). We establish necessary preliminaries about convergence of these zeta functions, and aboutcharacters on finite abelian groups. This will then enable us to prove Dirichlet's theorem.

References: Serre's "Course on Arithmetic" or Apostol's book onnumber theory.

3. Radhika Ganapathy: Introduction to Galois theory

Abstract: Given rational numbers b and c, we know that the roots of the quadratic polynomial $f(x) = x^2 + bx + c$ are given by $\frac{-b \pm \sqrt{b^2-4c}}{2}$. With a bit more work, similar formulas can be obtained for roots of cubic and quartic polynomials. However, it is impossible to obtain such formulas for a general n-th degree polynomial for $n \geq 5$. Galois introduced groups to study symmetries of roots of polynomials and gave a proof of the above mentioned impossibility statement. This lecture series, comprising of three lectures, will be an introduction to the subject of Galois theory.

4. Mahan Mj: Quotient Spaces

Abstract: We shall start with a general talk on the topological classification of surfaces (no background will be assumed), and note that the language of quotient spaces arises naturally in order to make precise the notions introduced in this talk. The remaining lectures will be devoted to identification spaces or quotient spaces, where we make sense of the notion of a topological space quotiented by an equivalence relation.

5. S.K. Roushon:Fundamental group and its applications

Abstract: In Topology, we study topological spaces and their properties which are invariant under 'equivalences'. Also, we ask when are two topological spaces 'equivalent'. We need to consider different kinds of equivalences, namely, homeomorphisms, homotopy equivalences, etc. Sometimes one can show that a weaker equivalence is easy to prove and gives the first step towards proving a stronger equivalence. Using General Topology one can solve such problems to a certain extent, but when spaces are

complicated, then we transfer the problem to some other areas of Mathematics, e.g., Group Theory. Many a time we see the problem is easy to solve in the new setup, and then trace back to the original problem to draw some conclusion. This is an extremely fruitful method and is done in the subject called Algebraic Topology.

In these lectures, we will start with well-known topological spaces and build the steps to solve some concrete problems, using the first important invariant called 'Fundamental Group' in Algebraic Topology. If time permits we will prove the existence of non-trivial knots in the Euclidean 3-space using fundamental groups.

The prerequisites are some basic General Topology. Topics like separations axioms, compactness, connectedness, retract, deformation retract and understanding the topology of Euclidean spaces and spheres (up to dimension 3). See standard book like, 'Topology' and 'Algebraic Topology' by Munkres. For knot theory, see the book by Crowell and Fox.

6. Swarnava Mukhopadhyay: Geometry of curves and surfaces

Abstract: This will be an introduction to the geometry of curves and surfaces lying in the 3-dimensional spaces.

We will discuss quantities like curvature and torsion for curves that measure how a curve twists in 3-dimensional space. We will also discuss the notions of Gaussian curvature and the mean curvature of surfaces. The former measure the intrinsic geometry of a surface while the latter describes the extrinsic geometry of how a surface bends in the ambient space. The celebrated Gauss-Bonet formula relates this geometric notion to the topology of the surfaces. Basic linear algebra and calculus are the only prerequisites for the course.

7. Sabyasachi Mukherjee:Analysis and geometry on the complex plane

Abstract: This minicourse will cover the essentials of analysis in one complex variable. We will start with basic definitions of holomorphic (or complex-analytic) maps, and discuss various characterizations of holomorphicity (such as integral representations, series expansions, etc.). A brief account of singularities of holomorphic maps, applications of the argument principle and residue theorem will also be given. In the last two lectures, we will touch upon geometric aspects of holomorphic maps. This will include conformality, the geometry of Möbius transformations, and the Schwarz lemma culminating in the description of the group of conformal automorphisms of the unit disk.

8. Mythily Ramaswamy: Integration in higher dimension

Abstract: After a brief review of continuity and differentiability of functions of several variables, we introduce integration and change of variable formula and a few applications. Then we take up line and surface integrals, followed by Stokes' theorem and the divergence theorem.

Special lectures in VV-2021, Mathematics
Title and Abstracts

1. Sushmita Venugopalan: Morse theory

Abstract: A manifold is a topological space that looks like Euclidean space near any point. Just as you define smooth functions on Euclidean space, one can also define smooth functions on manifolds. Given a typical smooth function on a manifold, one can learn a lot about the topology of the manifold using Morse theory.

2. Jaya Iyer: Lefschetz hyperplane section theorem

Abstract: We will discuss the topological aspects of spaces arising in Algebraic Geometry. More precisely we will introduce the concepts related to cell decomposition of projective spaces and compare with that of hypersurfaces.

3. Riddhi Shah: Certain Aspects of Dynamics

Abstract: We will start with an introduction to some aspects of dynamical systems followed by an overview of the dynamics of distal maps, which were introduced by David Hilbert. We will discuss distal automorphisms on locally compact groups and observe some properties. We will also characterise these automorphisms in terms of their contraction groups and behaviour of their orbits. Then we will briefly discuss the dynamics of billiards, a related illumination problem and related work of an inspiring woman mathematician.

Public Lecture by Rohini Godbole: Women in mathematics and physics through history

Date and Time: Monday, 5th July 2021, 5–6 PM

Abstract: I will begin by discussing why the scientific community needs to have more women in science. I will recount aspects of some of the prominent women of Mathematics and Physics beginning from the Greeks and ending with some 20 th Century women mathematicians, like Emmy Noether, Maryam Mirzakhani. In the case of Emmy Noether, I will discuss her life and the legacy of her work in some detail. This will be followed by a short discussion of what lessons we learn from these stories which are useful for today's women in science!