COURSE DETAILS

Week 1

Algebra 1: Linear representations of finite groups

Brief Overview. The goal of representation theory is to understand groups through their actions on vector spaces. This course will be an introduction to the representation theory of finite groups. Some of the topics covered will be irreducibility, Schur's lemma, complete reducibility, character theory, orthogonality relations and some applications.

Prerequisites. Group theory, quotients, normal subgroups, group actions on sets, orbit-stabilizer theorem, Sylow's theorems, solvable groups, Linear algebra, linear transformations, eigenvectors, Cayley-Hamilton, diagonalization, Jordan normal form, bilinear forms, Hermitian forms. Some familiarity with non-commutative rings (e.g. Group algebras) and modules over them will be helpful, but not strictly necessary.

Suggested references for the prerequisites: Abstract Algebra by Dummit and Foote for most of the topics except rings and modules. For Rings and Modules: Algebra by Lang (This is because the definition of rings and modules that we will use agrees with the one from Lang and not from Dummit and Foote, namely we will only work with unital rings and subrings of rings need to contain 1 etc.). Another reference: Algebra by Artin.

Analysis 1

To be announced.

Topology 1: Introduction to Topology

Brief Overview. On $\mathbb{R}^n$ or more generally on a set equipped with a distance function (a metric space), you have notions like open sets, closed sets, convergence, limit points, connected sets, continuous functions, ..., which help you to study the "geometry" of the space. But there are other interesting sets which might not naturally appear as metric spaces, and yet we want to study their "shape" and their "geometry". To do this we need to make sense of the above notions in a more general situation than metric spaces. This series of lectures will introduce some basic concepts in the subject of Topology, which forms the framework in which to study geometric Questions.

Prerequisites. Please revise concepts from your Real Analysis class relating to metric spaces. No specific text.

Week 2

Algebra 2: An introduction to the group structure of Elliptic curves

Brief Overview. In the language of abstract algebraic geometry, an elliptic curve is an abelian variety of genus 1. However in simpler words, it is just the zero set of a 'nice enough' cubic equation in two variables. Using elementary projective geometry, we will make the definition more precise and then describe a group law on such a curve. This binary operation gives structure of a finitely generated abelian group to the set of all rational points of an elliptic curve, which is a non-trivial theorem due to Mordell. If time permits, we will introduce few more invariants attached to an Elliptic curve and state some important theorems/ conjectures related to them. We will assume basic knowledge of algebra (groups, rings and modules, finite fields) and coordinate geometry.

Prerequisites. Coordinate geometry and basic algebra.

Analysis 2: An introduction to potential theory in the complex plane

Brief Overview. Originating in Newtonian physics, potential theory is a broad field of analysis concerning objects associated to the Laplace operator. In this course, we will study potential theory in the complex plane, where this subject is a blend of real and complex analysis. We will study the properties of harmonic functions, subharmonic functions, Green functions and potentials. We will discuss their connections to familiar objects in complex analysis. Finally, we will study Perron's method for solving the Dirichlet problem for the Laplacian on open sets in the plane.

Prerequisites. Familiarity with multivariable calculus (Chapter 9 in Rudin's Principles of Mathematical Analysis), complex analysis (Chapters III-VI in Conway's Functions of One Variable), and some basic measure theory (Chapters 1 and 2 in Folland's Real Analysis) will be helpful.

Topology 2

To be announced.