Assignment 0

(Submission deadline: 24.09.2020)

Exercise 1.

Show that a topological space *X* is Hausdorff if and only if the diagonal $\Delta(X) = \{(x, x) \in X \times X \mid x \in X\}$ is closed in $X \times X$.

Exercise 2.

Let $f : X \to Y$ be a surjective map of topological spaces. Show that the following are equivalent:

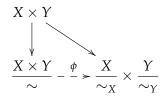
(a) *f* is a quotient map.

(b) a function $g: Y \to Z$ is continuous if and only if $g \circ f: X \to Z$ is continuous.

Exercise 3.

Let *X* and *Y* be topological spaces endowed with equivalence relations \sim_X and \sim_Y , respectively. Let \sim denote the equivalence relation on *X* × *Y* defined by setting $(x, y) \sim (x', y')$ if and only if $x \sim_X x'$ and $y \sim_Y y'$.

(a) Show that there is a unique bijective continuous map $\phi : \frac{X \times Y}{\sim} \to \frac{X}{\sim_X} \times \frac{Y}{\sim_Y}$ making the following diagram



commute, in which the vertical map is the quotient map and the diagonal map is the product of obvious quotient maps.

- (b) Give an example to show that ϕ need not be a homeomorphism.
- (c) If *Y* is locally compact and \sim_Y is the identity relation, then show that ϕ is a homeomorphism.

Exercise 4.

Let *X* and *Y* be topological spaces and let Y^X denote the set of all continuous functions $X \rightarrow Y$. Define a topology on Y^X by taking as a sub-basis all sets of the form

$$N_{K,U} = \{f : X \to Y \mid f(K) \subset U\},\$$

where *K* runs through all compact subsets of *X* and *U* runs through all open subsets of *Y*. This topology is called the *compact-open topology*.

- (a) Let *X* be locally compact and Hausdorff. Show that the *evaluation function* $e : Y^X \times X \to Y$ defined by e(f, x) = f(x) is continuous.
- (b) Let *X*, *Z* be Hausdorff and *Z* be locally compact. Show the *exponential law*: there is a natural function $Y^{Z \times X} \rightarrow (Y^Z)^X$, which is a homeomorphism.