Assignment 1

Exercise 1.

For pointed locally compact Hausdorff topological spaces (X, x_0), (Y, y_0) and (Z, z_0), show that the spaces $X \wedge (Y \wedge Z)$ and $(X \wedge Y) \wedge Z$ are homeomorphic.

Exercise 2.

Let (X, x_0), (Y, y_0) and (Z, z_0) be pointed topological spaces. Let $g_0 : Z \to Y$ denote the constant map y_0. Show that the exponential map $Y^{Z \times X} \to (Y^Z)^X$ induces a continuous function

$$(Y, y_0)^{(Z \times X, Z \vee X)} \to \left((Y, y_0)^{(Z, z_0)}, g_0 \right)^{(X, x_0)},$$

which is a homeomorphism if Z and X are Hausdorff and Z is locally compact.

Exercise 3.

Show that if (K, k_0) is an H-cogroup, then for every pointed topological space (X, x_0), the set

$$[(K, k_0), (X, x_0)]$$

can be given a natural group structure in the sense that every pointed map $f : (X, x_0) \to (Y, y_0)$ induces a group homomorphism $f_* : [(K, k_0), (X, x_0)] \to [(K, k_0), (Y, y_0)]$.

Exercise 4.

For any pointed topological space (X, x_0), show that the suspension ΣX is an H-cogroup.