Assignment 5

Exercise 1.

Show that the complement of any point $x \in \mathbb{RP}^2$ is homeomorphic to S^1. Use the Seifert-van Kampen theorem to show that $\pi_1(\mathbb{RP}^2, x) = \mathbb{Z}/2$.

Exercise 2.

For $P = (x_0, y_0) \in \mathbb{R}^2$, let X_P denote the shrinking wedge of circles $\cup_{n\in\mathbb{N}} X_{P,n}$ based at P, where $X_{P,n}$ denotes the circle centred at $(x_0 + 1/n, y_0)$ and of radius $1/n$. Show that the space

$$\widetilde{X} = \{0\} \times \mathbb{R} \ \cup \ \left(\cup_{n\in\mathbb{Z}} X_{(0,4n)} \right)$$

is a covering space of $X_{(0,0)}$. Construct a two-sheeted covering $Y \to \widetilde{X}$ such that the composite $Y \to \widetilde{X} \to X$ is not a covering.

Exercise 3.

Let G be a group and let $\mathcal{O}(G)$ denote its orbit category. Show that the function

$$Ob(\mathcal{O}(G)) \to Ob(\text{Top}); \quad G/H \mapsto X/H$$

extends to a functor $\mathcal{O}(G) \to \text{Top}$.

Exercise 4.

Consider S^1 as a subset of $\mathbb{R}^2 \times \{0\} \subset \mathbb{R}^3$. Show that $\mathbb{R}^3 \setminus S^1$ is homotopy equivalent to $S^1 \vee S^2$.