Assignment 6

(Submission deadline: 05.11.2020)

Exercise 1.

Let $n > 0$ be an integer and let Q_n be the closed subset of $\mathbb{CP}^{n+1}$ be the closed subset defined by the equation $x_0x_1 + x_2x_3 + \cdots + x_nx_{n+1} = 0$ if n is even and the equation $x_0x_1 + x_2x_3 + \cdots + x_{n-1}x_n + x_{n+1}^2 = 0$ if n is odd. Show that Q_n is a manifold of dimension n.

Exercise 2.

Let $k \leq n$ be positive integers. Show that $Gr_{\mathbb{C}}(k, n)$ is a compact connected Hausdorff manifold of dimension $2k(n-k)$.

Exercise 3.

Let M be a compact connected 2-dimensional manifold and let $M' = M \setminus \{x_1, \ldots, x_r\}$, for pairwise distinct points $x_1, \ldots, x_r$. Show that for any covering space $p' : N' \to M'$ with finitely many sheets, there exists a unique compact manifold N containing N' with $N \setminus N'$ a finite set of points such that p' extends to a map $p : N \to M$ (which need not be a covering map in general).

Exercise 4.

Let M be a connected Hausdorff manifold. Show that for any $x, y \in M$, there exists a homeomorphism $f : M \to M$ such that $f(x) = y$. Is it true that given any two pairwise distinct collections of points $x_1, \ldots, x_r$ and $y_1, \ldots, y_r$ for $r > 1$, there exists a homeomorphism $f : M \to M$ such that $f(x_i) = y_i$ for $1 \leq i \leq r$?