Assignment 7

Exercise 1.

Suppose that we have a pushout diagram of spaces

$$\begin{array}{ccc} A & \xrightarrow{i} & X \\ {\scriptstyle f}\downarrow & & \downarrow{\scriptstyle g} \\ B & \xrightarrow{j} & Y \end{array}$$

in which i is an inclusion of a subspace. Show that g induces a homeomorphism $X/A \to Y/B$.

Exercise 2.

Let $f : X \to Y$ and $g : Y \to Z$ be pointed maps. Show that $g \circ f$ is pointed nullhomotopic if and only if g has an extension $g' : \mathrm{hocofib}(f) \to Z$.

Exercise 3.

Let $f : X \to Y$ be a pointed map and let $j_f : Y \to \mathrm{hocofib}(f)$ and $j_{j_f} : \mathrm{hocofib}(f) \to \mathrm{hocofib}(j_f)$ denote the natural inclusions. Show that

$$\mathrm{hocofib}(j_{j_f})/C(\mathrm{hocofib}(j_f))$$

is homeomorphic to ΣY.

Exercise 4.

For any pointed map $f : X \to Y$, show that the quotient map

$$\mathrm{hocofib}(j_f) \to \mathrm{hocofib}(j_f)/CY \cong \Sigma X$$

is a homotopy equivalence.