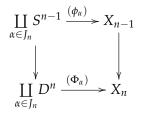
Assignment 9

Exercise 1.

Let *X* be a CW-complex and let X_n denote the *n*-skeleton of *X*, for $n \in \mathbb{N}$.

- (*a*) Show that X admits the colimit topology with respect to the family $\{X_n \mid n \in \mathbb{N}\}$.
- (*b*) Let $\{e_{\alpha} \mid \alpha \in J_n\}$ be the set of *n*-cells of *X*. Let Φ_{α} and ϕ_{α} denote the characteristic map and the attaching map of the cell e_{α} , respectively. Show that



is a pushout square in **Top**, where the vertical maps are natural inclusions.

Exercise 2.

- (*a*) Show that \mathbb{RP}^n is obtained from \mathbb{RP}^{n-1} by attaching one *n*-cell, for every $n \in \mathbb{N}$.
- (*b*) Show that \mathbb{CP}^n is obtained from \mathbb{CP}^{n-1} by attaching one 2*n*-cell, for every $n \in \mathbb{N}$.
- (c) Show that $S^{\infty} = \operatorname{colim}_n S^n$ is contractible.

Exercise 3.

Show that the inclusion of the 2-skeleton of a connected CW-complex X induces an isomorphism $\pi_1(X_2) \xrightarrow{\cong} \pi_1(X)$.

Exercise 4.

Give a CW-structure on a *torus with g-holes* (that is, connected sum of g copies of $T = S^1 \times S^1$), which has precisely 4g 0-cells, 4g 1-cells and a single 2-cell.