Assignment 9

Exercise 1.

Let X be a CW-complex and let X_n denote the n-skeleton of X, for $n \in \mathbb{N}$.

(a) Show that X admits the colimit topology with respect to the family $\{X_n \mid n \in \mathbb{N}\}$.

(b) Let $\{e_\alpha \mid \alpha \in J_n\}$ be the set of n-cells of X. Let Φ_α and ϕ_α denote the characteristic map and the attaching map of the cell e_α, respectively. Show that

$$\begin{array}{ccc} \coprod_{\alpha \in J_n} S^{n-1} & \xrightarrow{(\phi_\alpha)} & X_{n-1} \\ \downarrow & & \downarrow \\ \coprod_{\alpha \in J_n} D^n & \xrightarrow{(\Phi_\alpha)} & X_n \end{array}$$

is a pushout square in **Top**, where the vertical maps are natural inclusions.

Exercise 2.

(a) Show that $\mathbb{RP}^n$ is obtained from $\mathbb{RP}^{n-1}$ by attaching one n-cell, for every $n \in \mathbb{N}$.

(b) Show that $\mathbb{CP}^n$ is obtained from $\mathbb{CP}^{n-1}$ by attaching one $2n$-cell, for every $n \in \mathbb{N}$.

(c) Show that $S^\infty = \operatorname{colim}_n S^n$ is contractible.

Exercise 3.

Show that the inclusion of the 2-skeleton of a connected CW-complex X induces an isomorphism $\pi_1(X_2) \xrightarrow{\cong} \pi_1(X)$.

Exercise 4.

Give a CW-structure on a *torus with g-holes* (that is, connected sum of g copies of $T = S^1 \times S^1$), which has precisely $4g$ 0-cells, $4g$ 1-cells and a single 2-cell.