# Dynamical systems - Problem Set 5

(1) *Closing Lemma.*

   Let $f : X \to X$ be an open, continuous, distance expanding map on a compact metric space. Prove that for any $\varepsilon > 0$, there exists $\delta > 0$ such that any periodic $\delta$-pseudo-orbit $\{x_k\}_{k\in\mathbb{N}}$ of period $q$ (i.e., $x_{i+q} = x_i$, for all $i$) can be $\varepsilon$-shadowed by a periodic orbit of the same period.

(2) Let $f : \mathbb{R} \to \mathbb{R}$ be the function $f(x) = x^2$. Show that $HD(f(K)) = HD(K)$ for any $K \subset \mathbb{R}$.

(3) Let $K$ be the set consisting of the numbers between 0 and 1 whose decimal expansions do not contain the digit 5. Show that $HD(K) = \log 9/\log 10$.

(4) Let $A, B \subset \mathbb{R}^n$. Prove that $HD(A \times B) \geq HD(A) + HD(B)$.

(5) Prove that when $X$ is any subset of $\mathbb{R}^n$, and $K$ is the middle-thirds Cantor set, then
$$HD(X \times K) = HD(X) + HD(K).$$