Algebra 1
HW 3(Due: 07-09-2023)

1. What are the two sided ideals of $M_n(R)$ where R is any ring? Justify.

2. Let R be a ring and let D be a multiplicatively closed subset of R containing 1. Let $\phi : R \to D^{-1}R$ be the canonical homomorphism arising from localization. Give a necessary and sufficient condition for $D^{-1}R$ to be zero and for ϕ to be injective.

3. Show that $\mathbb{Z}[\frac{1+\sqrt{-19}}{2}]$ is not a Euclidean domain.

4. Use Gauss' lemma to show that R is UFD iff $R[x]$ is UFD

5. Show that $x^{n-1} + x^{n-2} + \cdots + 1$ is irreducible in $\mathbb{Z}[x]$ if and only if n is prime.

6. Show that $(\mathbb{Z}/2^r\mathbb{Z})^\times \simeq \mathbb{Z}_2 \times \mathbb{Z}_{2^{r-2}}$

7. Find all the abelian groups of order 270 and 320. For each of them list the invariant factors and elementary divisors.

8. Let $R = \mathbb{Z}[\sqrt{-n}]$ where $n > 3$ is squarefree.
   (a) Show that 2, $\sqrt{-n}$, $1 + \sqrt{-n}$ are irreducibles in R
   (b) Show R is not a UFD
   (c) Give an ideal of R that is not principal