

Algebra 1

HW 5(Due: 21-09-2023)

1. Solve Exercise 38, Section 12.3, Dummit and Foote
2. Exercises 14 Section 12.2, Dummit and Foote
3. Show that there are no finitely generated injective R -modules if R is a PID but not a field.
4. Show that if R is any integral domain and if I is a non-principal ideal of R , then I is torsion-free but not free as R -module
5. Let k be a field and \mathbf{Mat} be the category defined as follows: the objects are natural numbers and $Hom_{\mathbf{Mat}}(m, n) = \{n \times m \text{ matrices over } k\}$. Let \mathbf{FDVect} be the category of finite dimensional vector spaces. Show that \mathbf{Mat} and \mathbf{FDVect} are equivalent
6. Show that there is no functor $\mathcal{F} : \mathbf{Grp} \rightarrow \mathbf{Ab}$ that takes each group to its center. Here \mathbf{Grp} and \mathbf{Ab} are respectively the categories of groups and abelian groups. (Hint :Consider $S_2 \rightarrow S_3 \rightarrow S_2$ where S_n is the symmetric group in n letters)
7. Give an example of functors in each of the following cases
 - full but not faithful
 - neither faithful nor full
8. Prove that if $\mathcal{F} : \mathcal{A} \rightarrow \mathcal{B}$ is an equivalence of categories then \mathcal{F} is fully faithful and essentially surjective (In class we proved the other direction).