Algebra 1
HW 7 (Due: 05-10-2023)

1. Let $\mathcal{F} : \mathcal{C} \to \mathcal{D}$ and $\mathcal{G} : \mathcal{D} \to \mathcal{C}$ be such that $\mathcal{F} \dashv \mathcal{G}$. Prove the universal properties of the unit and the counit of the adjunction stated in the class.
2. Show that finite products (if they exist) coincide with finite coproducts in a preadditive category.
3. Give example of a monomorphism f in a category with objects (having underlying sets) such that f is not injective.
4. Let $\mathcal{F} : \mathcal{C} \to \mathcal{D}$ and $\mathcal{G} : \mathcal{D} \to \mathcal{C}$ be additive funbctors between abelian categories such that $\mathcal{F} \dashv \mathcal{G}$. Show that $\mathcal{F}$ is right exact and $\mathcal{G}$ is left exact.