1 Brief description of the course

The workshop is intended as an intensive introduction to the methods of algebraic geometry, concentrating on the important concepts, definitions and examples, with very few proofs. The lecturers will provide notes for their lectures which will give more details of proofs, and/or suitable references. The latter should help the participants devise their own detailed study programme in algebraic geometry, depending on the aspect of algebraic geometry they wish to pursue.

There will be 2 lectures per day, each of 90 minutes, and a discussion session to answer questions and resolve doubts, during the periods 15th-19th and 21st-24th April. The tentative plan of lectures is given below. In case of spillover the more advanced topics (towards the end) could be sacrificed.

2 Prerequisites

It is expected that the auditors will have had some exposure to commutative algebra, complex analysis, topology and differentiable manifolds as can be found (for example) in the following books.

1. Commutative algebra (references: books of Atiyah and Macdonald, or Gopalakrishnan; TIFR pamphlet).

2. Complex analysis (references: books of Ahlfors, Conway or Rudin (Real and Complex Analysis)).

3. Topology: fundamental groups, coverings, homology and cohomology (references: books of Massey, Greenberg, Dugundji, Spanier, or Vick).

4. Differential topology: calculus of several variables, Jacobians, implicit function theorem (references: books of Spivak (Calculus on Manifolds), or Rudin's Principles); differential manifolds, smooth maps, regular values, differential forms (references: first 2 chapters of Warner's book, or book of Singer and Thorpe, or Milnor's book (Topology from the differentiable viewpoint)).

3 Detailed contents

First week (15th April -19th April)	
1. Affine varieties; irreducible components; coordinate ring, Nullstellensatz; morphisms; Noether normalization, dimension; non-singularity and the Jacobian criterion; products.	$1'$. Differentiable manifolds, complex structures, symplectic forms, Riemannian metrics and Kähler manifolds. Complex projective n-space as a complex manifold. Fubini-Study metric. Complex submanifolds of $\mathbf{P}^n$. Review of smooth vector bundles.
2. Projective and quasi-projective varieties, homogeneous coordinate ring, basic affine open sets; irred. components; non-singularity; morphisms via linear systems (here, tuples of homogeneous polynomials of the same degree, or tuples of rational functions, satisfying suitable conditions); Segre and d-tuple embeddings; projective Noether normalization, degree; Hilbert-Samuel polynomial (defn., and statement that its leading coeff. determines degree).	$2'$. Recall that can define homology, cohomology, and some basic properties (e.g. finite generation); discuss Poincaré duality; discuss thm. that a compact R.S. is a non-singular projective algebraic curve.
3. Plane curves and space curves; divisor classes; Riemann-Roch theorem and applications. Plane curves of genus 0 and 1. Non-planarity of curves of genus 2. Ordinary double points and cusps; desingularization.	$3'$. Topology of plane and space curves. Differential forms and Riemann Hurwitz formula; genus formula. π_1 and uniformisation.
4. Grassmanians; dual varieties; Plücker embedding; More projective geometry: incidence; Chow form, Chow varieties; Schubert varieties and Schubert calculus; some enumerative problems.	$4'$. Hodge theory and Hodge-Lefschetz theory. Type decomposition of forms; Kähler structure; Hodge star, Laplacian; harmonic forms; statement of real Hodge theorem; statement that for Kähler, have 'complex' Hodge decomposition; Topology of affine varieties (Weak Lefschetz theorem). Hard Lefschetz theorem. Topology of hypersurfaces. Barth's theorem.
5. Ruled surfaces. Blowing up. Curves on ruled surfaces. Embeddings of ruled surfaces. Vector bundles on rank two on curves. Examples of nonruled surfaces. Holomorphic forms and a criteria for non-ruledness. Rationality. Quadrics and cubics.	$5'$. Morse theoretic proof of Weak Lefschetz. Topology of surfaces. K3 surfaces. Curve Fibrations over curves.

Second week (21th April -24nd April)	
6. Affine algebraic groups; tannakian theorem; interpreting representation theory in terms of geometry.	6′. Cohomology of sheaves; Čech cohomology; statement of Leray's theorem; relevant homological algebra(snake lemma, mapping cone); hypercohomology; de Rham's theorem in sheaf form.
7. Ringed spaces and schemes; locally free and invertible sheaves; quasi-coherent sheaves and modules; Serre's duality; Ext and Tor.	7′. Elliptic curves. algebraic and arithmetic theory. Moduli with level structures.
8,9. Additional topics: Semi-continuity and families of varieties; Hilbert Schemes and moduli. Picard groups in general (algebra and topology). Exponential sequence; Pic; Chern class for line bundles; Lefschetz theorem on (1,1) classes; Chern classes for vector bundles via curvature; cohomology class of an algebraic cycle; Hodge conjecture. Higher dimensional hypersurfaces. Cubic threefold. Quartic threefold and quintic threefold.	8′,9′. Abelian varieties. Theta functions. Singularities. Hironaka's theorem. Rational double points. Intersection multiplicity; Chow ring; algebraic Chern classes; Grothendieck's Riemann-Roch theorem.

References

[SGA 7 II] P. Deligne, N. Katz (eds.), *SGA 7 II: Groupes de Monodromie en Géométrie Algébrique*, Lect. Notes in Math. No. 340, Springer-Verlag (1973).

[F] W. Fulton, *Algebraic curves, An introduction to Algebraic Geometry*, Addison-Wesley (1989) (originally published 1969 by Benjamin in Math. Lect. Note Series).

[Fu1] W. Fulton, *On the topology of algebraic varieties*, in *Algebraic Geometry, Bowdoin, 1985*, Spencer J. Bloch ed., Proc. Symp. Pure Math. Vol. 46 Part 1, Amer. Math. Soc. (1987).

[GH] P. A. Griffiths, J. Harris, *Principles of Algebraic Geometry*, Pure and Applied Math., Wiley-Interscience (1978).

[H] R. Hartshorne, *Algebraic Geometry*, Grad. Texts in Math. No. 52, Springer-Verlag (1977).

[La] K. Lamotke, *The topology of complex projective varieties, after S. Lefschetz*, Topology, 20 (1981), 15-51.

[Lo] E. J. N. Looijenga, *Isolated singular points of complete intersections*, London Math. Lect. Note Ser. No. 77, Cambridge (1984).

[Mi] J. Milnor, *Morse Theory*, Annals of Math. Studies 51, Princeton, Fifth Printing, 1973.

[Mi2] J. Milnor, *Topology from the Differentiable viewpoint*, University of Virginia Press, 1976.

[Sw] R. Swan, *The Theory of Sheaves*, Chicago Lect. Notes in Math., Univ. of Chicago Press (1968).

[W] F. Warner, *Foundations of Differentiable Manifolds and Lie Groups*, Grad. Texts in Math. No.94, Springer-Verlag (1983).