Home Work #3 (Due: September 20,2025)

1. (Kempf)1.2.3, 1.2.4, 1.4.4 (following Cramer's rule as suggested), 1.5.5
2. (Kempf) 2.3.6, 2.3.7, 2.3.8, 2.3.9.
3. (Kempf) 3.1.6, 3.3.6, 3.3.7.
4. Consider the map $\nu : \mathbb{P}^1 \to \mathbb{P}^3$ given by the map $V \to Sym^3 V$, where V is a 2 dimensional vector space over $k = \overline{k}$. Let $F_0 = Z_0 Z_2 - Z_3^2$, $F_1 = Z_0 Z_3 - Z_1 Z_2$ and $F_2 = Z_1 Z_3 - Z_2^2$ and consider the twisted cubic C i.e. the zero locus of the above in $\mathbb{P}^3$.
   (a) Show that if $p \in C$, then either Z_0 or Z_3 has to non-zero and show that $p = \nu([Z_0, Z_1])$ or $p = \nu([Z_1, Z_3])$.
   (b) (Exercise 1.11 Harris) Consider $\lambda = [\lambda_0, \lambda_1, \lambda_2] \in \mathbb{P}^2$ and let Q_λ be the quadric defined by $F_\lambda = \lambda_0 F_0 + \lambda_1 F_1 + \lambda_2 F_2$. Show that for $\lambda \neq \nu$. The quadrics Q_λ and Q_ν intersect in C and a line $L_{\mu,\nu}$. This generalizes the example in class.
5. Read upto Theorem 1 in `https://people.reed.edu/~davidp/pcmi/laksov.pdf`. Don't stop if you find it interesting.
6. (Shafarevich)
   (a) Pages 53-54: 7-8-9-10-11.
   (b) Page 65-66: 2, 4, 7,8.
   (c) Read Section 6 Chapter I especially Tsen's theorem.
7. (Hartshorne, Pages 7-8) 1.1 (a,b), 1.2, 1.4, 1.6, 1.7, 1.10.
8. (Hartshorne, Pages 11-13) 2.4, 2.5., 2.6, 2.9, 2.10, 2.13, 2.15, 2.16.
9. (Hartshorne, Page 20-23) 3.1, 3.2, 3.3, 3.5, 3.9, 3.12, 3.13, 3.14