Exercise Sheet #4

(Due October 4)

1. Consider the Zariski topology of $\mathbb{A}^1$, and describe the closed subsets in the product topology of $\mathbb{A}^1 \times \mathbb{A}^1$.
2. Kempf: 3.7.3, 3.10.1
3. Hartshorne Exercises, Chapter 1.3: 3.17, 3.20
4. Read Example 2.4, 2.6, 2.7 from Harris and do Exercise 2.8, 2.9, 2.10 (Pages 23-25).
5. Show that if X be aclosed subvariety of $\mathbb{P}^{n_1} \times \dots \mathbb{P}^{n_k}$ and suppose all components of X have dimension $n_1 + \dots + n_k - 1$. Then X is defined by one equation that is homogenous in each of the k set of variables. (Shafarevich Page 69)
6. Show that any two curves in $\mathbb{P}^2$ meet. Is it true for any projective variety? Is $\mathbb{P}^2$ isomorphic to $\mathbb{P}^1 \times \mathbb{P}^1$?
7. If Q is the quadratic defined by the image of Segre map from $\mathbb{P}^1 \times \mathbb{P}^1$ in $\mathbb{P}^3$, show that there exists curves which are not defined by setting to zero a single form (i.e. a homogenous function on the cone) of $\mathbb{P}^3$.
8. Show that if $f : X \to Y$ is a map between projective varieties and all the fibers $f^{-1}(y)$ is irreducible (of same dimension) and Y is irreducible, then X is also irreducible.
9. As in class consider the map $\psi : \Gamma_m \to \Pi$. For any $y \in \Pi$, show that $\psi^{-1}(y)$ is irreducible of dimension $\frac{m(m+1)(m+6)}{6} - 1$. Using above conclude that Γ_m is irreducible.
10. Hartshorne Exercises, Chapter 2: 2.17 (Assume X and Y are irreducible varieties if schemes are involved).
11. If $f : X \to Y$ is affine (in the definiton of Kempf), is it true that for every affine open U in Y, the inverse image $f^{-1}(U)$ is also affine? Justify your answer.
12. Hartshorne Exercises, Chapter 3: 3.13, 3.17 (Assume X and Y are varieties if schemes are involved).
13. Read Proposition A.6 on page 287 in Shafarevich. If G is a finite group consisting of automorphism of a affine variety X and assume that the base field has characteristic zero. Show that $X \to Y = Spec(k[X]^G)$ is finite.