Algebraic Geometry
Fall 2025
Tuesday and Thursday 10:00 - 12:00, AG 77

Instructor: Swarnava Mukhopadhyay
Office: A-335
E-mail: swarnava@math.tifr.res.in
Course Website: `https://mathweb.tifr.res.in/~swarnava/teaching.html`

Course Descriptions: This is a course of lectures on "Algebraic Geometry" intended for second-year graduate students. After providing some basic motivations and examples, we will begin the course by examining algebraic curves and compact Riemann surfaces. We plan to cover topics in basic Riemann surface theory, like constructing Riemann surfaces from algebraic equations and constructions of differential forms and functions on them, focusing on the genus 1 case, Riemann extension theorems, analytic continuations, correspondence between covers of compact Riemann surfaces and their field extensions to construct meromorphic functions on Riemann surfaces. We will mostly follow the notes of D. Phong as well as the textbooks of S. Donaldson and O. Forster for this part.

We will then start exploring the notion of a space with functions and pursue algebraic varieties from this perspective. This will be the major part of the course. We will follow the books of D. Mumford and G. Kempf for these topics, which are a precursor to the abstract theory of schemes but with a lot of the essential features. After covering notions of affine and projective varieties, we will cover an array of interesting examples of varieties, affine and finite morphisms, blow-ups, dimension theory, notions of separateness, completeness, fibers, Chevalley's theorem, and Chow's lemma. After these essential topics, I will discuss the notion of sheaves and the notion of tangent bundles, cotangent bundles, and their holomorphic analogs, which appear in complex geometry from this perspective. We will cover sheaf cohomologies via various resolutions and their comparisons, the de Rham theorem. We will also discuss the notions of divisors and their correspondence with line bundles for complex manifolds. Once these essential tools and languages are developed, we will cover Weyl's lemma and Hodge decomposition for compact Riemann surfaces and give a proof of the Riemann-Roch theorem and Serre duality for compact Riemann surfaces.

Finally, we will introduce the notion of schemes (without repeating the old topics covered in the earlier part) and focus on the new technicalities, like generic points, specializations, the functor of points, gluing properties, and transferring the notion that we learnt previously for objects to morphisms between them. We will cover notions of coherent sheaves, sheaf of differentials, flatness,

This will be a pretty intense course, and your success depends mostly on the effort you put in. In particular, a lot of focus will be on solving the homework problems that will be assigned.

Text: We will not follow any specific text books for the entire length of the course. But here are some of the standard references that we will follow for various parts of the course.

- S. Donaldson, Riemann Surfaces.
- O. Forster, Lectures on Riemann Surfaces.
- P. Griffiths-J. Harris, Principles of Algebraic Geometry.
- R. Hartshorne, Algebraic Geometry.
- G. Kempf, Algebraic Varieties.
- D. Mumford, Red book of varieties and schemes.
- D. Phong, Lectures on Complex Analysis and Riemann Surfaces.

- I. Shafarevich, Algebraic Geometry I.

Homework and Grading: Regular problem sets will be posted after class on the website. It will very important that you work on those problems on a regular basis. We will have problem sessions, where you will be asked to present homework problems. Your grade in the course will depend on your class participation, presentation of homework problems and a very short in-person discussion at the end of the semester if required for some.

Classroom Etiquette: Please arrive in class on time. The room has only one door, and if you are late, it interrupts every possible thing. If you are late to class (unless due to an emergency), please spare us the interruption.
Students are expected to attend every class meeting with all communication devices turned off. Please make sure your cell phone does not ring during the lectures and distract others. Please do not use headphones or laptops during the class. I would like to encourage everybody to ask questions. Without intensive interactions, the lectures could feel uninteresting and monotonous. Maintaining a perfect learning atmosphere is a responsibility of every student in the class.