

Representation Theory of Lie algebras

Tuesday-Friday 10:30-12:00, Meeting online via Zoom

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Office Hours: By appointment on Zoom/Skype/Google Meet

Course Website: <http://math.tifr.res.in/~swarnava/August2020.html>

Text: I will not follow a particular book. I will suggest reference for various topics from a particular book as the course develops. There are some common references:

- *Lie Algebras and Lie Groups*, 1964 Lectures given at Harvard University by Jean-Pierre Serre.
- *Introduction to Lie Algebras and Representation Theory* by James E. Humphreys.
- *Representations of Compact Lie Groups* by Theoder Brocker and Tammo tom Dieck.
- *Infinite Dimensional Lie Algebras* by Victor. G. Kac.

Course Descriptions: The course is intended for second year Ph.D students. There are no prerequisites for this course except for the standard first year courses. I am planning to teach a two semester sequence on the topic. The second part is more geared towards applications and geometric methods. I plan to initially cover the following topics and then depending on time, we cover some topics from Kac-Moody algebras.

- Introduction to Lie groups and Lie algebras and basic properties and perspectives.
- Finite-dimensional semi-simple Lie algebras, complete reducibility, Killing forms, root decompositions, Weyl-group and root systems, classifications theorems, and Dynkin diagrams.
- Universal enveloping algebras, PBW theorem, Serre relations.
- Verma modules and representations of semi-simple Lie algebras, character formulas.
- BGG Category O.
- Representations of complex Lie groups, Borel-Weil theorem and some applications in invariant theory.
- Lie algebras without Serre relations, their Verma modules, and arrangements of hyperplanes.

Homework: Regular problem sets will be posted after class on the website. It will very important that you work on those problems on a regular basis.