Home Work 2

1. Chapter 3 Exercises 14, 15, 19, 22 (Stein and Shakarchi)
2. Chapter 3 Problem 1 (Stein and Shakarchi)
3. Suppose f_n is a sequence of holomorphic function in a connected domain Ω and $f_n \to f$ locally uniformly on Ω. Assume f_n is no where vanishing on Ω. Then f is either nowhere vanishing or identically zero.
4. Prove that for $\epsilon > 0$, the function $\frac{1}{z+i} + sinz$ has infinitely many zeros in the strip $|\operatorname{Im} z| < \epsilon$.
5. Let f be a homomorphic function in a neighborhood of a closed disc $\overline{D}$. Can the image of ∂D under f be the figure 8?
6. If $u \in C^2(\mathbb{C})$ is bounded and harmonic, then u is a constant.
7. Suppose f is an analytic in neighborhood of the closed unit disk D. Assume that f has finitely many zeros $z_1, \ldots, z_n$ in the interior of D with multiplicities $m_1, \ldots, m_n$. Show that

$$m_1 z_1 + \cdots + m_n z_n = \int_{\partial D} \frac{z f'(z)}{f(z)} dz$$