# Analysis II

Tuesday-Thursday 10:00-11:30, AG-77

**Office:** School of Mathematics, A335
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**Office Hours:** After class or by appointment.
**Course Website**: `https://mathweb.tifr.res.in/~swarnava/Jan2024.html`

**Text:** I will try to follow the topics of the book on *Complex Analyis* by E. Stein and R. Shakarchi for the first part of the course. For the remaining part of the course I will cover miscellenous topics depeding on the time and interest. Ideally in the second part, I would like to cover some aspect of the theory of *Riemann Surfaces and theta functions.* I will suggest references for various topics from particular books as the course develops. You are expected to attend all the classes, take notes, actively participate, and contribute to an excellent learning environment.

There are some common references:

- *Complex Analysis* by E. Stein and R. Shakarchi.
- *Complex Analysis* by L. Ahlfors.
- *Lectures on Riemann Surfaces* by O. Forster.
- *Lecture Notes on Riemann Surfaces* by S. Donaldson
- *Complex Analysis in one variable* by R. Narasimhan.
- *TIFR Pamphlets on Riemann Surfaces* by M. S. Narasimhan, R. R. Simha, R. Narasimhan and C. S. Seshadri.

**Course Descriptions:** This is the second analysis course for all first year mathematics students entering the Ph.D or Int-Ph.D. program in TIFR. This course along with analysis one will be part of the interview which is conducted at the end of the first year of the program. We will cover the following:

- Local theory of holomorphic functions and Cauchy Riemann equations
- Cauchy's integral formula, Goursat's theorem, Morera's theorem
- Open mapping theorem, fundamental theorem of algebra.
- Meromorphic functions, residues formula, analysis of zeros and poles, essential singularities.
- Hadamard and Weierstrass factorization theorems, Mittag Leffler theorem, Runge's theorem.
- Logarithms and branch cuts, residue calculus and various applications.
- Analytic continuations, Gamma functions, Riemann zeta functions and their functional equations.
- Differential equations with regular singularities with focus on explicit examples.
- Schwarz's lemma, Normal families, Montel's theorem and Riemann mapping theorem.
- Automorphisms of upper half plane.
- Basics of Riemann surfaces following Donaldson's book.
- Riemann surfaces associated to explicit polynomial equations, normalization of plane curves.

- Riemann surfaces associated to $w = \sqrt{z(z-1)(z-\lambda)}$ and its function theory
- Riemann surface structures on tori and Weierstrass theta functions
- Jacobi theta functions, Abel-Jacobi type theorem and existence of meromorphic forms and function on tori with specified properties from three different viewpoints.
- Discussion on Riemann-Roch theorems and uniformization of Riemann surfaces

**Grading Scheme:** Regular problem sets will be posted on the website regularly and will be graded. Homework will be due at the **beginning** of the class on the due date. No late homework will be accepted unless previously agreed upon or an emergency.

Let me mention a word or two about the homework. It will very important that you work on all problems on the homework set and also the ones suggested in the class. There would be multiple problems on the homework and you are supposed to work and submit all of them (unless specified). However **only a few (say 3-4) chosen (by me after you submit)** will be graded. Feel free to discuss problems amongst your batchmates but the writing should be on your own.

The grading scheme for the class will be the following:

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| Homework | 20 % |
| Midterm | 25 % |
| Presentation | 15 % |
| Final Exam | 40 % |

The final exam will be a **cumulative exam** covering all the topics covered in the class. The topics for presentation will be announced in a few weeks and will take place between the midterm and the final exam.