

MODULI PROBLEMS AND ALGEBRAIC STACKS

- **Lecture 1:** This lecture will be an introduction to various moduli problems, their history and a functorial view point. It will focus on some known obstructions on the existence of the solution to these moduli problems and provide some motivation for considering stacks.

We will mostly follow the material from Chapter 1 of Jarod Alper's notes as well as some examples from Newstead's book. (Swarnava)

- **Lectures 2, 3, 4 :** We will focus on two explicit moduli problems, namely the moduli problem of classifying pointed stable curves up to isomorphism and moduli problem of classifying bundles on a smooth projective curve. We will discuss some features and constructions in and around those moduli problems. We will also state the result on existence of Hilbert and Quot schemes (Swarnava)
- **Lecture 5:** In this lecture, we will discuss moduli problems with automorphisms and introduce groupoids. We will use this to motivate and discuss the notion of prestacks (Charanya)
- **Lecture 6:** We will discuss why étale topology is relevant in the set-up of moduli problems and progress with discussing Grothendieck topologies, sites, pre-sheaves, sheaves and sheafification on sites. We will also discuss various aspects of Descent theory. (Charanya)
- **Lectures 7, 8 :** We will discuss some explicit examples of representable functors like the Grassmannians, Hilbert Schemes and Quot Schemes and outline their constructions. (two students)
- **Lectures 9, 10:** We will introduce algebraic spaces and stacks, covering definitions of algebraic spaces, Deligne-Mumford stacks and algebraic stacks. We will put these in the context of the moduli problems discussed earlier (Charanya)
- **Lecture 11:** First properties of algebraic spaces and stacks, Equivalence relations and groupoids of schemes (student)
- **Lecture 12:** Representability of the diagonal (student)
- **Lecture 13:** Dimension, tangent spaces, and residual gerbes

- **Lectures 14:** Moduli of bundles on curves: Introduction(Swarnava)
- **Lecture 15-16:** Details on constructing Bun_G (two students)
- **Lecture 17:** Uniformization theorem(student)
- **Lecture 18:** Line bundles on moduli-“determinant of cohomology” (student).
- **Lecture 19:** We will discuss various characterizations of Deligne-Mumford stacks, smoothness and the formal lifting criterion, properness and the valuative criterion. We will roughly cover Chapter 3.6-3.9 of Alper’s notes.
- **Lecture 20:** Geometry of Deligne-Mumford stacks, elaborated on separated and properness, and their valuative criteria. Quasi-coherent sheaves on Deligne-Mumford stacks, Local structure theorem for Deligne-Mumford stacks
- **Lecture 21:** Keel Mori theorem, existence of coarse moduli spaces.
- **Lecture 22-27:** Moduli of curves, universal families .

REFERENCES