

Exercise Sheet #2

1. Let k be field not necessarily algebraically closed and G be an algebraic group scheme over k .
 - (a) Show that G is separated.
 - (b) Prove that G is reduced if and only if G is smooth.
 - (c) Show that the reduced subscheme G_{red} underlying G is a smooth subscheme of G .
2. Consider the problem of classifying $(n \times n)$ matrices over an algebraically closed field k upto conjugation. Determine the open orbits and closed orbits and the orbits closures of each orbits.
3. Define the Hopf algebra structure on $k[x]$ corresponding to the algebraic group \mathbb{G}_a .
4. Define the Hopf algebra structure on $k[\mathrm{GL}_n(k)]$ corresponding to the algebraic group $\mathrm{GL}_n(k)$.