Exercise Sheet #3

For all these problems assume that the base field is algebraically closed of arbitrary characteristics.

1. If G is a finite group of order not divisible by $\mathrm{char}(k)$, then $\underline{G}$ is linearly reductive over k.

2. Show that $\mathbb{G}_a$ is not reductive. Hint: Consider the obvious faithful representation of $\mathbb{G}_a$ to GL_2

3. Compute the X^s for the action of GL_n on $\mathrm{Mat}(n)$ by conjugation.

4. Consider the space of $\mathbb{A}^3$ identified as the space of degree 2 homogeneous polynomials of degree 2 in variables x and y. Consider the standard action of SL_2 on $\mathbb{C}^2$ with coordinates (x, y). This induces an action on $\mathbb{A}^3$. Compute the ring of invariants.

5. Show that $k[\mathrm{Mat}_n]^{\mathrm{GL}_n}$ is $k[c_1, c_2, \ldots, c_n]$, where c_i's are coefficients of the characteristic polynomial. Hint: Use the closed orbits description.