Exercise Sheet #5

1. Consider the action of $t \in \mathbb{G}_m$ on $k[x_0, \ldots, x_n]$ by $(x_0, \ldots, x_n) \to (t^{-1}x_0, tx_1, \ldots, tx_n)$. We know that $X//\mathbb{G}_m$ is $\mathbb{P}^{n-1}$. Compute the pull back of $\mathcal{O}_{\mathbb{P}^{n-1}}(1)$ to $\mathbb{P}^n$ under the quotient morphism φ.

2. Consider the action of SL_2 on the space of 2×2 matrices over an algebraically closed field of characterestic zero. Extend the action to $\mathbb{P}^4$ in an obvious way with a linearlization. Show that the GIT quotient is $\mathbb{P}^1$ and compute $\varphi^* \mathcal{O}_{\mathbb{P}^1}(1)$ to $\mathbb{P}^4$. under the quotient morphism q.

3. Do the same for the first example in Exercise Sheet 4.