Exercise Sheet #6

1. Let $f : X \to \mathbb{P}(V)$ be a G-equivariant map, where G acts on $\mathbb{P}(V)$ via its linear representation. Show that $L = f^*(\mathcal{O})(1)$ admits a G-linearization and the map f is the map given by the line bundle L.

2. If X is connected and complete, give an explicity description of the Kernel of the forget full map $\mathrm{Pic}^G(X) \to \mathrm{Pic}(X)$.

3. Show that any line bundle on a normal irreducible variety X on which SL_n acts admits a unique SL_n-linearization.

4. Let G be a geometrically reductive group acting on $\mathbb{A}^n$ and 0 in the orbit closure of a point $z \in \mathbb{A}^n$. Show that we can find a smooth projective curve C and a rational map $p : C \to G$ and a k-point $c_0 \in C$ such that $\lim_{c \to c_0} p(c)z = 0$. (Hint find a curve C_2 in G that dominates the curve C_1 constructed in Step I of the proof in class)