

Exercise Sheet #7

1. Show that μ -stability implies Gieseker stability.
2. Let X be a smooth projective curve and \mathcal{E} and \mathcal{F} are locally free sheaves on X of rank r_1 and r_2 . Show that

$$\det \mathcal{E} \otimes \mathcal{F} \simeq \det \mathcal{E}^{\otimes r_2} \otimes \det \mathcal{F}^{\otimes r_1}$$

3. The following proves that the Grassmannian variety $Gr^{quo}(r, d)$ represents the functor $Quot_{\mathcal{O}_{\mathbb{Z}}/\text{Spec } \mathbb{Z}}^{d, \mathcal{O}}$. We break it into the following steps:
 - (a) Show by mimicing the proof of Problem 1 in Exercise Sheet 1 construct $Gr^{quo}(r, d)$ exists as a schemes over $\text{Spec } \mathbb{Z}$.
 - (b) Show that $Gr^{quo}(r, d) \rightarrow \text{Spec } \mathbb{Z}$ is a separated. (It is also proper and one check it via the valuative criterion)
 - (c) Construct the universal quotient over $Gr^{quo}(r, d)$.