Exercise Sheet #8

1. Let $\mathcal{E}$ be a coherent sheaf over $S \times X$ which is flat over S and suppose that for some $(s, x) \in S \times X$, the sheaf $\mathcal{E}_s$ is locally free at x. Then show that the cohorent sheaf $\mathcal{E}$ is locally free at (s, x). Show that this is equivalent to the following:

   (a) If A and B are Noetherian local rings and m be the maximal ideal of A and $\phi : A \to B$ is a local homomorphism and let E be a finite B-module which is flat over A. Then, if E/mE is free over B/mB, E is free over B.

   (b) Use Nakayama's lemma twice to give a proof.

2. Use the previous problem to show that $R' = \{q \in \text{Quot} \mid \mathcal{U}_q \text{ is locally free}\}$ is open in Q.

3. Show that $R^{(s)s}$ is open in R'. (Hint: Use Semicontinuity Theorem methods to show that higher direct images are locally free.)