Home Work 4

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1. Show that the even part of the center of the Clifford algebra $C(Q)$ is just $\mathbb{C}$.
2. Let $S^+ = \bigwedge^{ev} W$ and $S^- = \bigwedge^{odd} W$ be the Spin-representations of $\mathfrak{so}(2m)$. Show that if for any two subset I and J of $\{1, \ldots, m\}$, we can find elements in the Lie algebra $\mathfrak{so}(2m)$ that takes e_I to e_J provided $|I| - |J|$ is even.
3. Prove that the center of the Special Clifford Group $\mathrm{SC}(Q, \mathbb{C})$ is $\mathbb{C}^*$.
4. Using the norm map show that $\mathrm{SC}(Q, \mathbb{C}) \simeq \mathrm{Spin}(n, \mathbb{C}) \times_{\mathbb{Z}/2} \mathbb{C}^*$.
5. Show that the Kernel of the natural map $\mathrm{SC}(Q, \mathbb{C}) \to \mathrm{SO}(n)$ has kernel $\mathbb{C}^*$.
6. Prove the following exceptional isomorphism
   (a) $\mathrm{Spin}(6) \cong \mathrm{SL}(4)$.
   (b) $\mathrm{Spin}(3) \cong \mathrm{SL}(2)$.
   (c) $\mathrm{Spin}(4) \cong \mathrm{SL}(2) \times \mathrm{SL}(2)$.