Home Work 9

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1. Problems 1,2,10,11,12,14 from Humphreys Page 54-55.
2. Problems 1,2,3,5,7,11 from Humphreys Page 40-41. For problems 1,2,3 only do it for $\mathfrak{sl}(n)$ and $\mathfrak{sp}(2n)$.
3. Finish the proof of the conjugacy of BSA's as follows: Assume that $\mathfrak{t} \subsetneq \mathfrak{h}$ is strict.
   (a) Consider the case $\mathfrak{b}' \subset C_{\mathfrak{g}}(\mathfrak{t})$. Find a Borel subalgebra $\mathfrak{b}''$ in $C_{\mathfrak{g}}(\mathfrak{t})$ containing $\mathfrak{h}$ and conjugate to $\mathfrak{b}$ to arrive at a conclusion.
   (b) Now consider the case $\mathfrak{b}'$ is not contained in $C_{\mathfrak{g}}(\mathfrak{t})$.Find a common eigen vector $x \in \mathfrak{b}'$ for $ad\mathfrak{t}$ and an element $t \in \mathfrak{t}$ such that $[t, x] = x$.
   (c) Let $\mathfrak{s} := \mathfrak{h} \oplus_{\alpha \in \Phi_t} \mathfrak{g}_\alpha$, such that Φ_t is the set of roots β such that $\beta(t)$ is a positive rational number. Show that $\mathfrak{s}$ is solvable
   (d) Let $\mathfrak{b}''$ be a BSA contained $\mathfrak{s}$. Show that $\mathfrak{b}'' \cap \mathfrak{b}$ has strictly higher dimension that $\mathfrak{b}' \cap \mathfrak{b}$.
   (e) Use the above and the induction hypothesis to show that $\mathfrak{b}'$ is conjugate to $\mathfrak{b}$.