Math 808-Geometric Invariant Theory

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Course Descriptions: The aim of the course is to study moduli problems in algebraic geometry and the construction of moduli spaces via geometric invariant theory. Taking quotient of a space by the action of a group is a fundamental operation in geometry. One is interested in cases where the quotient spaces lands up in the same category (for example Topological manifolds, complex manifolds, C^∞-manifolds)as the original space itself. Geometric Invariant theory is a theory of takings quotients in algebraic geometry.

A moduli problem is a classification problem, where we have a class of objects we want to classify up to some equivalence relation; for example, hypersurfaces in a projective space up to the automorphisms of the projective space or vector bundles on a variety up to isomorphism. In the course, we will study moduli functors, algebraic groups and their actions, affine quotients and projective quotients by algebraic groups, criteria for semistability, as well as some classical moduli problems and their solutions using GIT. Below is the detailed description of topics covered.

Topics Covered: We will try to cover the following topics depending on time and interest of the participants.

- What is GIT and what are the issues in taking quotients in algebraic geometry ? Brief discussion o Categories, Functors and Yoneda Lemma. Detailed discussion of what is a moduli problems ? Examples of fine moduli spaces, coarse moduli spaces and how we can use GIT in moduli problems. The main references for this section are [FGK+05, Mum99].
- Affine Quotients and Projective Quotients with many elementary examples. Criterion for Stability (Hilbert-Mumford Criterion). Many examples of calculation of stability conditions will be discussed. This is one of the main goals of this course. The main references for this section are [New78, MFK94].
- Moduli Problems: Construction of the moduli of vector bundles over a curve of fixed rank and degree $\mathcal{M}(r,d)$. Picard group of $\mathcal{M}(r,d)$. Construction of the moduli space of curves of genus g. [Sim94a, Sim94b, New78, LP97, Gie82]
- Variation of GIT for quotients of Toric varieties. The main references for this section are [Muk03, Tha94].

Course Website: The website is `http://www2.math.umd.edu/~swarnava/topicsgit.html`.

References

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[Muk03] Shigeru Mukai, *An introduction to invariants and moduli*, Cambridge Studies in Advanced Mathematics, vol. 81, Cambridge University Press, Cambridge, 2003. Translated from the 1998 and 2000 Japanese editions by W. M. Oxbury. ↑(document)

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